

# **Order in space**

A.K. van der Vegt

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## **PREFACE**

This book deals with a very old subject. Many centuries ago some people were already fascinated by polyhedra, and they spent much time in investigating regular spatial structures. In the terminology of polyhedra we, therefore, meet the names of Archimedes, Pythagoras, Plato, Kepler etc.

Today, polyhedra play a role in crystallography, art, architecture, and, in particular, in hobbyistic mathematics: many people are still so much delighted by them, that they cannot leave from constructing, analyzing and playing with polyhedra, and, above all, enjoying them.

This book is the result of several decades of these activities, step by step carried out in spare time. It is intended to stimulate fellow-hobbyists to a further interest in this fascinating subject !

Delft, February 2001

A.K.van der Vegt



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