

Numerical Methods for Ordinary Differential Equations

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C. Vuik P. van Beek F. Vermolen J. van Kan

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Preface

In this book we discuss several numerical methods for solving ordinary differential equations. We emphasize those aspects that play an important role in practical problems. In this introductory text we confine ourselves to ordinary differential equations with the exception of the last chapter in which we discuss the heat equation, a parabolic partial differential equation. The techniques discussed in the introductory chapters, for e.g. interpolation, numerical quadrature and the solution of nonlinear equations, may also be used outside the context of differential equations. They have been included to make the book self contained as far as the numerical aspects are concerned. Chapters, sections and exercises marked * are not part of the Delft Institutional Package.

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C. Vuik

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