Dynamic data processing

recursive least-squares

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Foreword

This book is based on the lecture notes of the course *Dynamic data processing* as it has been given by the Department of Mathematical Geodesy and Positioning (MGP) of the Delft University of Technology since 1990. The prerequisites are a solid knowledge of adjustment theory and geodetic positioning, together with linear algebra, statistics and calculus. The theory and application of least-squares adjustment are treated in Adjustment theory (Delft University Press, 2000). The material of the present course extends the theory to the recursive estimation of time-varying or dynamic parameters. The time-varying parameters could for instance be geometric parameters such as position, attitude and shape, physical parameters such as temperature and humidity, or instrumental parameters such as clock drifts and biases. The time-varying parameters are said to be determined recursively when the method of determination enables sequential, rather than batch processing of the measurement data. The main goal is therefore to convey the knowledge necessary to be able to process sequentially collected measurement data in an optimal and efficient manner for the purpose of estimating time-varying parameters.

Following the *Introduction*, the basic theory of least-squares estimation is reviewed in *Chapter 1*. This is done for the model of observation equations and for the model of condition equations. In *Chapter 2* the principle of recursive least-squares estimation is introduced. The recursive principle allows one to update the least-squares solution for new observations without the need to store all past observations. Two different forms of the measurement-update equations are given. The results of Chapter 2, which hold true for time-invariant parameters, are generalized in *Chapter 3* to the case of time-varying parameters. The time-varying nature of the parameters is assumed captured by means of polynomial equations of motion. The recursive solution now consists of two types of update equations, the measurement-update equations and the time-update equations. Since there still exist many dynamic systems for which the rather simple polynomial model of Chapter 3 does not apply, a larger class of dynamic models is introduced in *Chapter 4*. These models are formulated using the state-space description of dynamic systems. In order to include randomness in the state-space description of dynamic systems, some of the elementary concepts of the theory of random functions are discussed in *Chapter 5*. This chapter also includes a description of the propagation laws for linear, time-varying systems. The results of Chapter 5 are used in *Chapter 6* to model possible uncertainties associated with the dynamic model. As a result the update equations are obtained for the recursive least-squares filtering and prediction of time-varying parameters.

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