Chapter 15

CRUISE PERFORMANCE

15.1 Range and endurance

As depicted in Figure 15.1, here, the term range is used for the horizontal straight-line distance an airplane travels in cruising flight, whereas the distance traversed in climb, cruise, and descent is called *total range*, *stage length* or *block distance*. Maximum total range is the distance an airplane can fly between takeoff and landing as limited by its fuel capacity. The fuel consumption per unit time is

$$F = \frac{dW_f}{dt}, \tag{15.1}$$

where W_f is the total fuel load.
Since $dW_f = -dW$, the fuel weight flow rate is related to the weight of the airplane by (see also Chapter 8)

$$F = -\frac{dW}{dt}. \tag{15.2}$$

The range is obtained from the following definite integral,

$$R = \int_{t_1}^{t_2} V\,dt = \int_{W_1}^{W_2} -\frac{V}{F}dW = \int_{W_2}^{W_1} \frac{V}{F}dW, \tag{15.3}$$

where V/F is the specific range (range per unit weight of fuel). The subscripts ”1” and ”2” refer to the initial and final conditions at start and end of cruise, respectively.

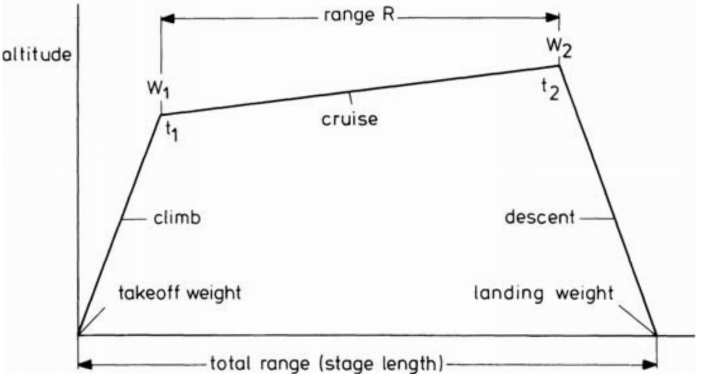

Figure 15.1 Mission nomenclature

The term *endurance* is used for the length of time spent in cruising flight. The endurance can be written as

$$E = \int_{t_1}^{t_2} dt = \int_{W_1}^{W_2} -\frac{dW}{F} = \int_{W_2}^{W_1} \frac{dW}{F}. \tag{15.4}$$

At this point it is important to remember that in symmetric flight, the time history of the flight condition depends on the specification of two control laws, that is to say, the description of the variation of two control variables with time (see Chapter 8).

Generally, both control variables are held constant throughout the cruise so that the flight condition only changes due to the influence of fuel consumption on airplane weight.

For airplanes propelled by airbreathing engines, however, there is only a slow variation of airplane weight. This observation allows us to consider the flight as a continuous succession of uniform motions under slowly varying conditions. In other words, the instantaneous values of V/F and F can be determined as though the airplane is in quasi-steady-state flight.

The procedure for determining F and V/F as a function of airplane weight may be illustrated by reference to Figures 15.2a and 15.2b for a propeller-driven and a jet-driven airplane, respectively. In both cases it is assumed that the airplane is performing a level flight at constant engine control setting.

In the case of propeller propulsion, the problem requires the computation of the level flight speed at a number of airplane weights from the equilibrium condition $P_a = P_r$. To each flight velocity, there corresponds a particular value of propulsive efficiency η_j and specific fuel consumption c_P. Then, successive engine powers can be found by using Equation (6.1),

$$P_{\text{br}} = \frac{P_a}{\eta_j}. \tag{15.5}$$

The corresponding fuel weight flow rates can be computed from Equation (6.15),

$$F = c_P P_{\text{br}}. \tag{15.6}$$

Analogously, for the jet picture we may use the respective points of intersection of the thrust and drag curves to give the successive values of airspeed V, thrust T, and specific fuel consumption c_T. According to the definition of c_T, Equation (6.54), the fuel weight flow rates can be obtained from

$$F = c_T T. \tag{15.7}$$

Now, Equations (15.3) and (15.4) can be evaluated graphically to give the values of range and endurance for the chosen cruise technique of constant altitude and constant engine control setting.

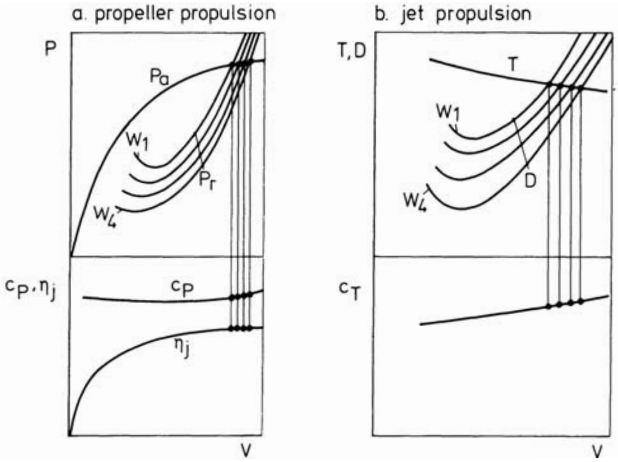

Figure 15.2 Determination of V/F and F (flight at constant altitude and engine control setting)

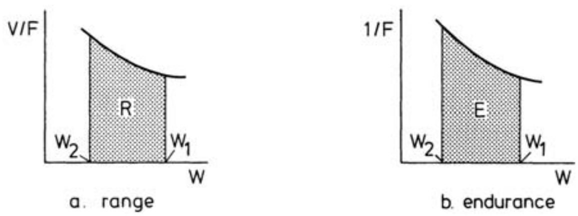

Figure 15.3 Calculation of range and endurance

Range follows from plotting V/F against W, as shown in Figure 15.3a. The shaded area under the curve from the final cruise weight W_2 to the initial cruise weight W_1 represents the range. Similarly, the endurance follows from the plot of $1/F$ versus W, as shown in Figure 15.3b.

If the preceding calculation procedure is done systematically a so-called *cruise chart* can be deduced as exemplified in Figure 15.4. The diagram gives for a specified altitude the typical variations of specific range with airspeed and airplane weight, showing the various cruise techniques.

In Figure 15.4, the constant engine rating program is represented by the line AB, the constant speed program by the line AC, and the maximum range program by the line AD, all for the same initial cruising speed. Of these, the most realistic cruise program is to maintain a constant airspeed at a fixed altitude (line AC). In this connection, it may be remarked that, practically, the range achieved by this cruise technique is virtually the same as that gained by the maximum range program (Reference 50).

The instantaneous airspeeds which give the greatest distance on a given quantity of fuel (line AD), may be called maximum range speeds or economic speeds, V_{ec}. Obviously, for a particular cruise program, the relationship between specific range

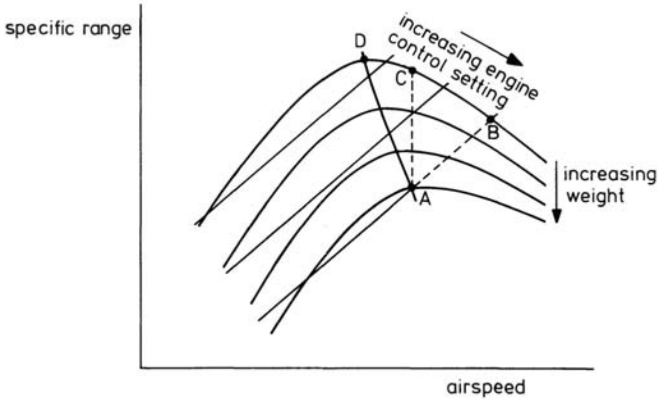

Figure 15.4 Typical specific range performance at a specified altitude

and airplane weight follows directly from the chart and so the resulting range for a given amount of fuel.

15.2 Approximate analytic expressions for range and endurance (propeller propulsion)

To obtain analytic expressions for range and endurance, we note that specific range and fuel weight flow rate can be related to the characteristics of the airplane and propulsion system by using Equations (15.5) and (15.6). Assuming quasi-level and quasi-steady flight, we can write

$$F = c_P P_{\text{br}} = c_P \frac{P_a}{\eta_j} = c_P \frac{P_r}{\eta_j} = c_P \frac{DV}{\eta_j}. \tag{15.8}$$

Making use of the relationships of Chapter 9 that $V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$ and $D = \frac{C_D}{C_L}W$, we obtain

$$\frac{V}{F} = \frac{\eta_j}{c_P}\frac{C_L}{C_D}\frac{1}{W} \tag{15.9}$$

$$F = \frac{c_P}{\eta_j} W \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{C_D^2}{C_L^3}}. \tag{15.10}$$

Substituting Equation (15.9) into Equation (15.3), and Equation (15.10) into Equation (15.4) gives

$$R = \int_{W_2}^{W_1} \frac{\eta_j}{c_P}\frac{C_L}{C_D}\frac{dW}{W} \tag{15.11}$$

$$E = \int_{W_2}^{W_1} \frac{\eta_j}{c_P} \frac{dW}{W\sqrt{\frac{W}{S}\frac{2}{\rho}(C_D^2/C_L^3)}}. \tag{15.12}$$

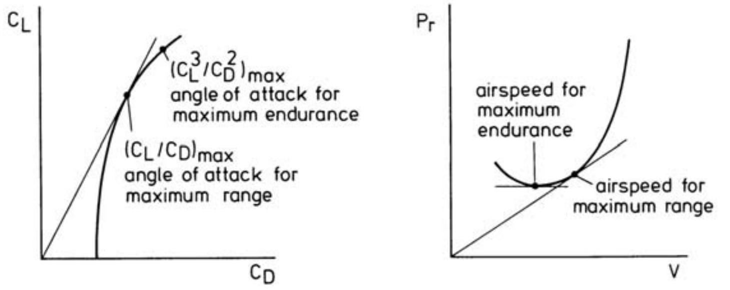

Figure 15.5 Best range and endurance conditions in level flight for propeller-driven airplanes

An examination of Equations (15.11) and (15.12) reveals that from an analytical point of view it is interesting to consider the cruise technique where the angle of attack is held constant throughout the flight. Furthermore, η_j and c_P usually exhibit only small variations over the band of cruising speeds so that it is possible to assume that they have constant average values.
Then, Equation (15.11) can be integrated to give an approximate analytic expression for the range,

$$R = \frac{\eta_j}{c_P}\frac{C_L}{C_D}\int_{W_2}^{W_1}\frac{dW}{W} = \frac{\eta_j}{c_P}\frac{C_L}{C_D}\Big|\ln W\Big|_{W_2}^{W_1} = \frac{\eta_j}{c_P}\frac{C_L}{C_D}\ln\frac{W_1}{W_2}. \tag{15.13}$$

This expression is the classic Breguet formula for range, derived by the Frenchman Louis-Charles Breguet (1880-1955).
Inspection of Equation (15.13) learns that to maximize range, flight must be carried out at the angle of attack at which C_L/C_D is the maximum. This is the flight condition for minimum airplane drag (Figure 15.5).
Also note that Equation (15.13) can be used for both constant altitude and climbing flight. If the altitude is kept constant, then we see from the relationship $W = C_L\frac{1}{2}\rho V^2 S$ that the airspeed must be steadily reduced as fuel is consumed. On the other hand, if the airspeed is held constant, the cruising height must be gradually increased during the course of the flight. Therefore, the latter cruise technique is commonly referred to as cruise-climb flight. It should be remarked that this type of flight may not be tolerable in many situations because of the requirements of air traffic control (A.T.C.).
For propeller airplanes powered by piston engines, the term η_j/c_P in Equation (15.13) remains nearly constant when airspeed or altitude are changed. Consequently, there will be no difference in range when the two cruise programs are compared; only an increase in cruising speed for the cruise-climb flight.
For turboprop airplanes, however, high cruising altitudes are essential, since at a given engine rating the specific fuel consumption decreases with height. Also it is important that at the economic airspeed the engines operate at their maximum

permitted cruise rating in order to achieve the lowest possible specific fuel consumption. Obviously, the only way to attain the optimum cruising condition is to fly at a high altitude.

To obtain a closed form solution for the endurance we shall consider the realistic cruise technique of a flight at constant altitude. Then, carrying out the integration of Equation (15.12), with η_j and c_P assumed constant throughout the flight, produces the following formula

$$E = \frac{\eta_j}{c_P}\sqrt{\frac{C_L^3/C_D^2}{\frac{1}{S}\frac{2}{\rho}}}\int_{W_2}^{W_1}\frac{dW}{W\sqrt{W}} = \frac{\eta_j}{c_P}\sqrt{\frac{C_L^3/C_D^2}{\frac{1}{S}\frac{2}{\rho}}}\left|\frac{-2}{\sqrt{W}}\right|_{W_2}^{W_1}$$

$$= \frac{\eta_j}{c_P}\sqrt{\frac{C_L^3/C_D^2}{\frac{1}{S}\frac{2}{\rho}}}\left[\frac{2}{\sqrt{W_2}} - \frac{2}{\sqrt{W_1}}\right]. \tag{15.14}$$

Introducing the airspeed at the starting point of the cruising flight, that is

$$V_1 = \sqrt{\frac{W_1}{S}\frac{2}{\rho}\frac{1}{C_L}}, \tag{15.15}$$

we modify Equation (15.14) to obtain

$$E = 2\frac{\eta_j}{c_P}\frac{C_L}{C_D}\frac{1}{V_1}\left[\sqrt{\frac{W_1}{W_2}} - 1\right]. \tag{15.16}$$

Combination of Equations (15.13) and (15.16) yields the following expression for the average velocity during the flight

$$V_{av} = \frac{R}{E} = \frac{V_1 \ln(W_1/W_2)}{2(\sqrt{W_1/W_2} - 1)}. \tag{15.17}$$

Equation (15.14) indicates that for best endurance, the airplane must fly at the angle of attack at which C_L^3/C_D^2 is the maximum. This is the flight condition for minimum power required and minimum fuel weight flow rate.
Inspection of Figure 15.5 shows that level flight speeds less than the speed for best endurance are in the region of reversed command. As was demonstrated in Section 11.3, flying in this region introduces the problem of speed instability. Because of this phenomenon the actual cruising speed lies somewhat above the minimum power required speed, e.g., $V_{cr} \geq 1.1V_{D\min}$ (cf. Equation (11.13)).

15.3 Approximate analytic expressions for range and endurance (jet propulsion)

Assuming quasi-steady level flight and using the relationship $D = \frac{C_D}{C_L}W$, the thrust can be written as

$$T = D = \frac{C_D}{C_L}W. \tag{15.18}$$

With Equation (15.7) and the relationship $V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$, the specific range is found to equal

$$\frac{V}{F} = \frac{1}{c_T W}\sqrt{\frac{W}{S}\frac{2}{\rho}\frac{C_L}{C_D^2}}. \tag{15.19}$$

Substituting Equation (15.19) into Equation (15.3) yields

$$R = \int_{W_2}^{W_1} \frac{1}{c_T W}\sqrt{\frac{W}{S}\frac{2}{\rho}\frac{C_L}{C_D^2}}\,dW. \tag{15.20}$$

The corresponding integral, expressing the endurance is obtained by insertion of Equations (15.7) and (15.18) into Equation (15.4),

$$E = \int_{W_2}^{W_1} \frac{1}{c_T}\frac{C_L}{C_D}\frac{dW}{W}. \tag{15.21}$$

In deriving analytic expressions for range and endurance, first, we shall consider cruising at a fixed height and at a constant angle of attack. Moreover, we shall continue to assume that the specific fuel consumption remains constant for the duration of the flight. The analysis will be further simplified by neglecting the variation of the effects of compressibility on the aerodynamic characteristics of the airplane as the flight speed reduces during the course of the flight.
Integrating Equation (15.20), we find

$$R = \frac{1}{c_T}\sqrt{\frac{2}{S\rho}\frac{C_L}{C_D^2}} \int_{W_2}^{W_1} \frac{dW}{\sqrt{W}} = \frac{2}{c_T}\sqrt{\frac{2}{S\rho}\frac{C_L}{C_D^2}}\,\left|\sqrt{W}\right|_{W_2}^{W_1}$$

$$= \frac{2}{c_T}\sqrt{\frac{2}{S\rho}\frac{C_L}{C_D^2}}\left[\sqrt{W_1}-\sqrt{W_2}\right]. \tag{15.22}$$

It should be remarked that $\sqrt{\rho}$ is present in the denominator of Equation (15.22), and that this is the essential reason why high cruising altitudes are desired for jet powered airplanes. By using Equation (15.15), we can rewrite Equation (15.22) as follows,

$$R = \frac{2}{c_T}\sqrt{\frac{W_1}{S}\frac{2}{\rho}\frac{C_L}{C_D^2}}\left[1-\sqrt{\frac{W_2}{W_1}}\right] = 2\frac{V_1}{c_T}\frac{C_L}{C_D}\left[1-\sqrt{\frac{W_2}{W_1}}\right], \tag{15.23}$$

where V_1 is the initial airspeed.
Performing the integration of Equation (15.21) gives

$$E = \frac{1}{c_T}\frac{C_L}{C_D}\int_{W_2}^{W_1}\frac{dW}{W} = \frac{1}{c_T}\frac{C_L}{C_D}\left|\ln W\right|_{W_2}^{W_1} = \frac{1}{c_T}\frac{C_L}{C_D}\ln\frac{W_1}{W_2}. \tag{15.24}$$

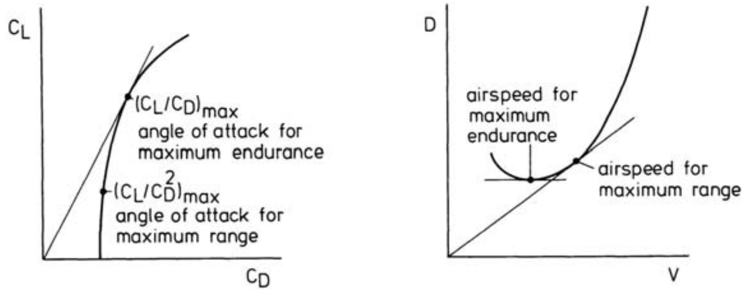

Figure 15.6 Best range and endurance conditions in level flight for jet-powered airplanes

From Equations (15.23) and (15.24) the average airspeed during the flight is found to be

$$V_{av} = \frac{R}{E} = \frac{2V_1(1-\sqrt{W_2/W_1})}{\ln(W_1/W_2)} \tag{15.25}$$

An inspection of Equations (15.22) and (15.24) shows that:

- For best range, the airplane should be flown at the angle of attack for maximum C_L/C_D^2. From the relationships $V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}}$ and $D = \frac{C_D}{C_L}W$, it may be seen that this requirement corresponds to the flight condition for minimum D/V (Figure 15.6).
- At a given angle of attack, range increases with altitude up to the typical cruising altitude. The favorable effect of a higher altitude is augmented by the tendency of the specific fuel consumption to decrease with increasing altitude up to the tropopause (I.S.A.).
- Maximum endurance will be obtained when C_L/C_D is the maximum. This is the flight condition for minimum airplane drag (Figure 15.6).

A second cruise technique of interest for turbojet and turbofan airplanes is the flight at constant airspeed and angle of attack. For this cruise technique, we pointed out in Section 15.2 that as fuel is burned, the airplane should ascend in altitude.
The flight-path angle occurring in this cruise-climb schedule, however, is normally sufficiently small so as to approve the use of the level-flight conditions that lift is equal to weight and thrust is equal to drag.
From Equations (15.3), (15.7) and (15.18), we then have

$$R = \int_{W_2}^{W_1} \frac{V}{c_T}\frac{C_L}{C_D}\frac{dW}{W}. \tag{15.26}$$

If again c_T and C_L/C_D are assumed to have constant values throughout the flight, Equation (15.26) can be readily integrated to give the expression

$$R = \frac{V}{c_T}\frac{C_L}{C_D}\ln\frac{W_1}{W_2}, \tag{15.27}$$

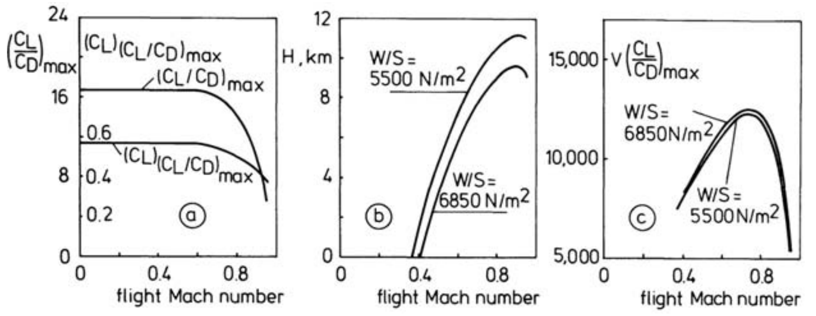

Figure 15.7 Condition for maximum $V(C_L/C_D)$

where the quantity $\frac{V}{c_T}\frac{C_L}{C_D}$ is called the *range factor*.

Sometimes it may be convenient to express the range in terms of the overall efficiency of the propulsion system. According to Equation (6.55), we have

$$\eta_{\text{tot}} = \frac{gV}{Hc_T}. \tag{15.28}$$

Insertion of Equation (15.28) into Equation (15.27) gives

$$R = \eta_{\text{tot}} \frac{H}{g} \frac{C_L}{C_D} \ln \frac{W_1}{W_2}. \tag{15.29}$$

As mentioned earlier in Chapter 6, the heating value H of all common aviation fuels (hydrocarbon fuels) is about 4.3×10^7 Joule/kg so that the ratio H/g in Equation (15.29) is about 4390 km.
Equations (15.27) and (15.29) are also labeled Breguet range equations, although Breguet's name was originally associated with range performance of airplanes driven by the combination of piston-engines and propellers. In this light, it is worthwhile to remark that Equation (15.29) also holds for propeller-driven airplanes. This statement can easily be verified by substitution of Equations (6.16) and (6.17) into Equation (15.13).
At constant airspeed and angle of attack, the endurance is directly found as

$$E = \frac{R}{V} = \frac{1}{c_T} \frac{C_L}{C_D} \ln \frac{W_1}{W_2}. \tag{15.30}$$

Obviously, the greatest endurance will be obtained when C_L/C_D is the maximum. Also note that Equation (15.30) is identical to the expression for the endurance in level flight (see Equation (15.24)).

Equation (15.27) indicates that for a given initial weight and fuel load, the airplane should fly at that altitude and airspeed at which the product $V(C_L/C_D)$ is a maximum, provided that variations in specific fuel consumption can be neglected.

The derivation of this flight condition will be demonstrated from a numerical example. For this, we return to our illustrative turbofan airplane with its Iift-drag polars given in the previous Figure 9.7. From this data, the curves in Figure 15.7a are deduced, which give the maximum lift-to-drag ratio and the associated lift coefficient as a function of flight Mach number. The graph manifests the characteristic behavior that both quantities drop off sharply at Mach numbers greater than 0.6. In Figure 15.7b are plotted the required altitudes (I.S.A.) versus M as computed from the relationship $W = C_L \frac{1}{2}\gamma p M^2 S$ where C_L is the lift coefficient at which C_L/C_D is the maximum. The related airspeeds follow from $V = Mc$.
The final result is shown in Figure 15.7c, in the form of a plot of the product $V(C_L/C_D)_{\max}$ against flight Mach number. Clearly, there is an optimum flight Mach number and an optimum altitude at a given airplane weight. Figure 15.7b also demonstrates that as the weight of the airplane decreases during cruise the airplane should climb in altitude to maintain the optimum flight condition.

Note from Equation (15.27) that with the assumptions of constant c_T and C_L/C_D, the range has no absolute maximum. Without compressibility drag, a constrained optimum is obtained when the magnitude of the airspeed is specified. In this case, the maximum range will also occur when C_L/C_D is the maximum. This condition requires that the instantaneous height should be that height at which the minimum drag speed becomes equal to the chosen airspeed.
When our cruise-climb flight is conducted in the lower stratosphere (I.S.A.), where the speed of sound is constant, a fixed airspeed also means a fixed flight Mach number. Consequently, the aerodynamic ratio in Equation (15.27) is exactly constant (see Chapter 4). If we look at the force equation

$$T = C_D \frac{1}{2}\rho V^2 S, \tag{15.31}$$

we see that the thrust is directly proportional to the air density. However, at a constant value of T/ρ, turbo-engine performance in the lower stratosphere (I.S.A.) is such that engine control setting is fixed. As a result, also specific fuel consumption remains unchanged (see Chapter 6). Therefore, the desired flight program is realized if the pilot simply maintains constant readings on the Mach meter and the engine-speed indicator. The appropriate expression for the range is then obtained by substituting Equation (15.31) into Equation (15.27). The resulting form becomes

$$R = \frac{1}{c_T}\sqrt{\frac{T}{S}\frac{2}{\rho}\frac{C_L^2}{C_D^3}}\ln\frac{W_1}{W_2}. \tag{15.32}$$

We observe that the condition for maximum range when flying in the lower stratosphere (I.S.A.) at a given engine control setting and airspeed exists when C_L^2/C_D^3 is the maximum.

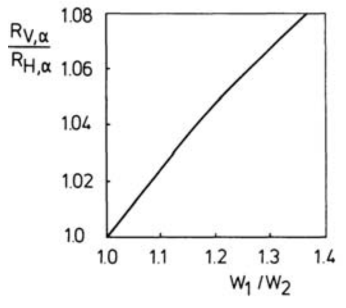

Figure 15.8 Relative range

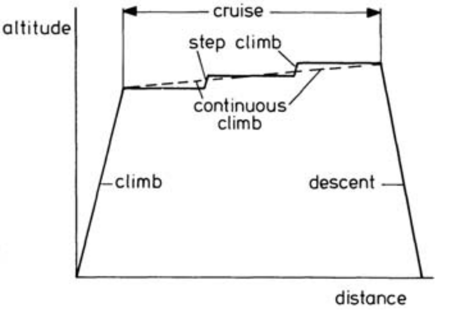

Figure 15.9 Stepped altitude flight

A third constrained optimum is derived for a specified altitude at the starting point of the cruise-climb flight. Then, insertion of Equation (15.15) into Equation (15.27) leads to the expression

$$R = \frac{1}{c_T}\sqrt{\frac{W_1}{S}\frac{2}{\rho_1}\frac{C_L}{C_D^2}}\ln\frac{W_1}{W_2}, \tag{15.33}$$

where ρ_1 is the air density at the initial height H_1.
Examination of the above equation reveals that to obtain maximum range, flight must be executed at maximum C_L/C_D^2.
Taking the initial height of the cruise-climb flight to be the cruising height of the constant altitude flight and assuming the same angle of attack and specific fuel consumption, we can derive a simple relationship between the two ranges. Dividing Equation (15.27) by Equation (15.23) yields

$$\frac{R_{V,\alpha}}{R_{H,\alpha}} = \frac{\ln(W_1/W_2)}{2(1-\sqrt{W_2/W_1})}. \tag{15.34}$$

The above ratio is presented in Figure 15.8. The graph indicates that the increase in range by performing a cruise-climb flight becomes greater as the weight ratio W_1/W_2 increases. In other words, flying cruise-climb appears to be more economical than level flight, especially when the airplane executes a long-distance flight. In practice, a cruise-climb flight may be approximated by a sequence of level flight segments (Figure 15.9). However, when the cruise is executed under the jurisdiction of flight traffic control regulations, each altitude change needs approval. Then cruising altitudes as well as cruising speeds and headings are assigned by air traffic control in order that sufficient spacing is ensured vertically, longitudinally, and laterally for safe flight.

15.4 Effect of wind on cruise performance

In this section we shall pay some attention to the effect of wind on cruising performance. For simplicity, we shall consider the presence of a constant headwind or tailwind only, directed along the flight path.

From our analysis in Chapter 14, we know that if an airplane flies in a steady wind, the motion is governed by the same equations as used in still air. Thus, drag and power required curves are unaffected although now ground speed differs from airspeed.

In level and quasi-level flight, the relationship between ground speed V_g, airspeed V, and wind velocity V_W, is given by (cf. Equation (1.24))

$$V_g = V - V_W, \tag{15.35}$$

where, according to our sign convention, a headwind is taken positive and a tailwind negative.

Wind does not affect the endurance of an airplane because it is a function only of fuel consumption per unit time. On the contrary, wind has a pronounced effect on range.

The specific range with respect to the ground can be expressed as

$$\frac{V_g}{F} = \frac{V - V_W}{F} = \frac{V}{F}\left[1 - \frac{V_W}{V}\right]. \tag{15.36}$$

Equation (15.36) indicates that the with-wind and zero-wind specific ranges are different. For example, when $V = V_W$ the distance traveled relative to the ground is zero. Clearly, the presence of wind also affects the economic speed. This can be investigated by considering the following relationships:

$$\frac{V_g}{F} = \frac{V_g}{c_P P_{\text{br}}} = \frac{\eta_j}{c_P}\left[\frac{V - V_W}{P_r}\right]. \tag{15.37}$$

$$\frac{V_g}{F} = \frac{V_g}{c_T T} = \frac{1}{c_T}\left[\frac{V - V_W}{D}\right]. \tag{15.38}$$

Note that Equations (15.37) and (15.38) concern the specific range of propeller-driven airplanes and jet powered airplanes, respectively.

As shown in Figure 15.10, with wind the economic speeds with respect to the ground are found by determining new origins on the airspeed axes and drawing tangents to the power required curve and the drag curve. From the constructions shown, it is seen that for both airplane types the airspeed for maximum specific range with a headwind is greater than in still-air conditions. The reverse is true when flying with a tailwind. Further, it may be understood from Figure 15.10 that maximum specific range, and so the maximum range, is increased by a tailwind and decreased by a headwind.

An expression for the range relative to the ground with a wind of velocity V_W is given by

$$R = \int_{W_2}^{W_1} \frac{V_g}{F} dW = \int_{W_2}^{W_1} \frac{(V - V_W)}{F} dW = \int_{W_2}^{W_1} \frac{V}{F} dW - V_W \int_{W_2}^{W_1} \frac{dW}{F} \quad \text{or}$$

$$R = R_{(V_W=0)} - V_W E. \tag{15.39}$$

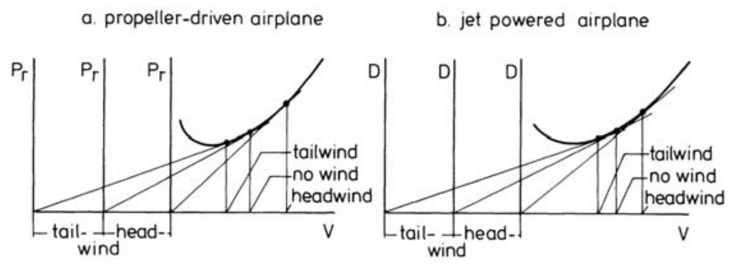

Figure 15.10 Effect of steady headwind and tailwind on airspeed for maximum specific range

Again this equation shows that range is affected advantageously by a tailwind and adversely by a headwind. Therefore, it will be clear that the actual fuel load of the airplane not only is determined by the flight distance but also by the prevailing winds along the flight path. These may be known from meteorological forecasts, completed with seasonal wind data for the airway.

15.5 Weight breakdown

The total weight of the airplane W_{to}, may be written as the sum of the structural weight W_c, the weight of the propulsion system W_e, the payload W_P, the fuel weight W_f, and the weight of the reserve fuel W_{fr}. Thus,

$$W_{to} = W_c + W_e + W_P + W_f + W_{fr}. \tag{15.40}$$

The total weight is the weight at takeoff brake release (TOW), and depends on the loading condition. TOW should not exceed the maximum takeoff weight (MTOW), which is generally determined by structural considerations.

The reserve fuel must be onloaded above the trip fuel to provide for changes in the intended flight profile or flight program and for diversion to an alternate airport due to a balked landing at the airport of destination. The amount of reserve fuel is usually determined by the operator in accordance with operational procedures. Typically, the procedure allows for a flight to alternate and a standard stacking time (Figure 15.11).

Payload is the weight of passengers and cargo. The sum of the payload and trip fuel may be called the useful load W_u.

$$W_u = W_P + W_f. \tag{15.41}$$

The structural weight will include not only the weight of the airframe but also the weight of fixed and removable equipment, furnishings, and the weight of the complete crew. The structural weight and the weight of the propulsion system may be combined into the operational empty weight (OEW) or basic operational weight W_b,

$$W_b = W_c + W_e. \tag{15.42}$$

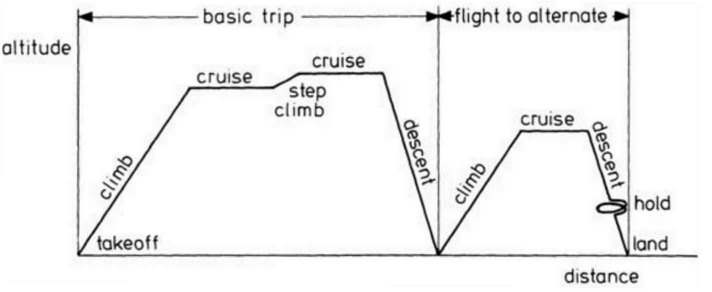

Figure 15.11 Typical flight profile

Thus, W_b is the weight of the airplane fully equipped excluding only payload and fuel,

$$W_{to} = W_b + W_p + W_f + W_{fr}. \tag{15.43}$$

If we divide through by the total weight, we obtain the weight breakdown in terms of weight fractions,

$$1 = \frac{W_b}{W_{to}} + \frac{W_p}{W_{to}} + \frac{W_f}{W_{to}} + \frac{W_{fr}}{W_{to}}. \tag{15.44}$$

Because of their usefulness, weight fractions are often employed in airplane performance and design considerations. For example, when we assume that the entire journey length is performed in cruising flight, from Equations (15.27) and (15.29), the ratio W_f/W_{to} can be written as

$$\frac{W_f}{W_{to}} = 1 - e^{-\frac{R}{\frac{V}{c_T}(C_L/C_D)}} = 1 - e^{-\frac{R}{\eta_{tot}\frac{H}{g}(C_L/C_D)}}, \tag{15.45}$$

where now R is the total range. Equations (15.45) and (15.44) show that the aim is obviously to make the range factor as large as possible and to keep the fraction W_b/W_{to} low in order to obtain a large payload fraction. The weight fraction W_b/W_{to} may be regarded as the structural efficiency since the lighter the airplane is built, the greater is the useful load fraction.

Typical weight fractions for a stage length of 6500 km are shown in Figure 15.12 for a high-subsonic turbofan airplane and a supersonic transport with turbojet engines. Due to its lower range factor, the fuel fraction for the supersonic airplane is higher than for the turbofan airplane. Consequently, the payload fraction for the supersonic transport is considerably smaller, notwithstanding its higher structural efficiency.

We also conclude from Equation (15.45) that at a given range factor, a greater fuel fraction is required as the range becomes longer. This implies that for a given airplane the payload fraction decreases with range as can be seen from Equation (15.44).

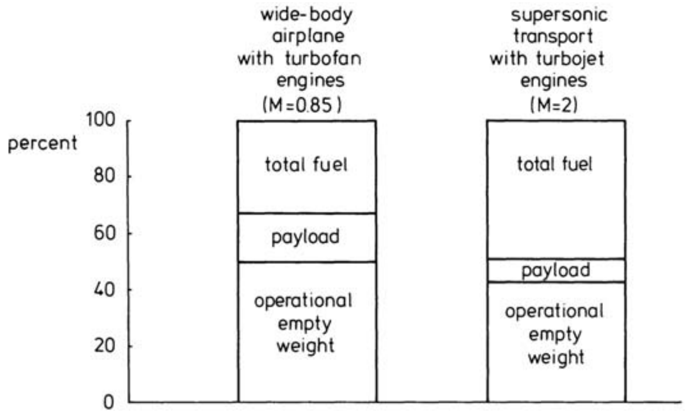

Figure 15.12 Typical weight breakdowns

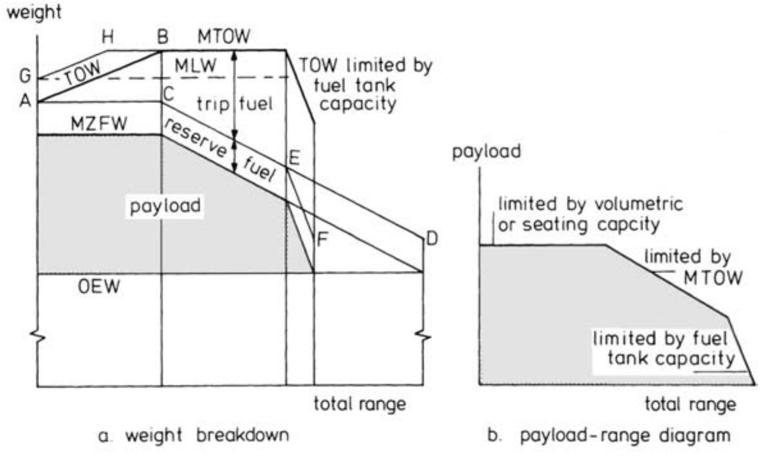

Figure 15.13 Payload-range characteristics

In Figure 15.13a the weight breakdown is sketched with respect to stage length in still air for a typical transport airplane. The line AB gives the takeoff weight at the maximum payload that can be carried. Point B corresponds to the maximum stage length with maximum payload. This range is called the design range. Increasing the total range above the design range requires that payload is replaced by fuel. This is represented by the line CD. At point D the ultimate range is reached (zero payload and reserve fuel unconsumed). Usually, the fuel tank capacity is such that the range cannot be increased beyond point E. The line EF, finally, indicates that some further increase of the total range is possible by reducing the takeoff weight when consuming the full fuel load.

The line GH in Figure 15.13a represents a limit to the takeoff weight that may be dictated on short ranges by the maximum allowable weight of the airplane at the landing. The maximum landing weight (MLW) is imposed by structural design requirements. Another weight limit is the maximum zero fuel weight (MZFW), the maximum allowable weight of the airplane without fuel. Normally, the maximum landing weight is greater than the maximum zero fuel weight plus the reserve fuel. Otherwise, payload might be restricted by the limited strength of the landing gear or airframe structure under particular landing conditions.

The payload-range relationship of Figure 15.13a is separately portrayed in Figure 15.13b. The latter curve may be regarded as the basis of the economic value of a transport airplane.

15.6 The economic performance of transport airplanes

In this section are summarized the most valuable parameters determining the commercial merit of a transport airplane. These performance items are given below and will be explained in the order listed:

- block time, E_B
- block speed, V_B
- transport product, P_R
- transport productivity, P_h
- revenue-earning capacity, P_y

The *block time* is the total time elapsing from starting engines at the departure airport to engines off at the destination place. Thus, the block time includes taxi time from the loading point to the takeoff runway, checks, takeoff, ascent to cruising height, cruise, descent, final circuits, approach and landing, and taxi time to the terminal point.

The *block speed* is the block distance divided by the block time,

$$V_B = \frac{R}{E_B}. \tag{15.46}$$

Evidently, the block speed is lower than the cruising speed. According to Reference 51, the relationship between block time, block distance and cruising speed can be written as

$$E_B = \frac{R}{V_{cr}} + \Delta t, \tag{15.47}$$

where R is the block distance, V_{cr} is the cruising speed and Δt is the length of time that accounts for the field operations and the lower airspeeds in flight phases other than the cruise.

Combining Equations (15.46) and (15.47) results in the following expression for the block speed,

$$V_B = \frac{R}{\frac{R}{V_{cr}} + \Delta t}. \tag{15.48}$$

Typical variations of block time and block speed with total range are plotted in Figure 15.14, using Δt = 50 minutes. The graphs show that at a given cruising

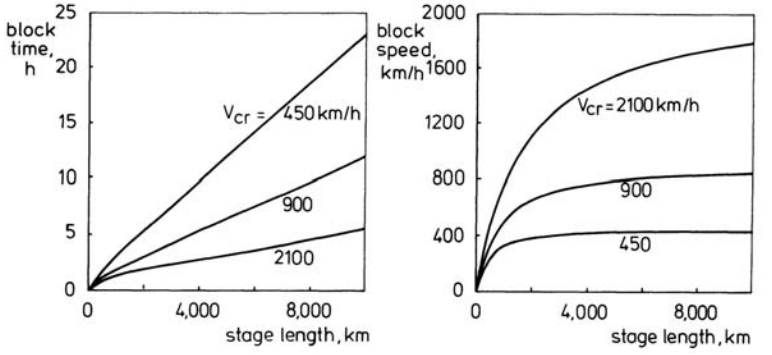

Figure 15.14 Block time and block speed versus range

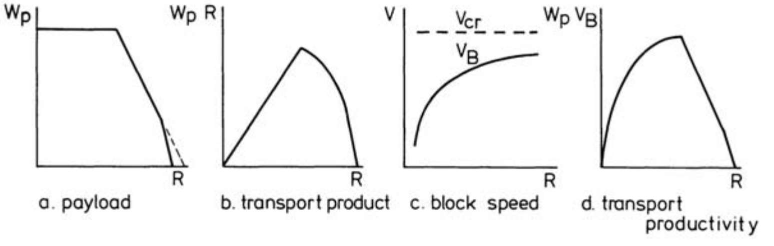

Figure 15.15 Economic parameters for transport airplane

speed both block time and block speed increase with increasing block distance, and that raising the cruising speed is more beneficial as the range is greater.
Obviously, the revenues which are gained by transport of passengers and/or freight are dependent on payload as well as range. Therefore, the transport performance is given by the product of payload and range,

$$P_R = W_p R, \tag{15.49}$$

where P_R is named the *transport product* and may be expressed in the units tonkm or passengerkm.
For a transport airplane with payload-range characteristic as depicted in Figures 15.13b and 15.15a, the variation of the transport product versus stage length is as shown in Figure 15.15b. It is of interest to note that the peak value of the transport product occurs at a flight distance which is equal to half the ultimate range. In the case that the latter distance is shorter than the design range, the optimum value occurs at the design range. Also note that the revenues are directly proportional to the load factor, the ratio of the payload actually carried over a given route distance to that payload that could have been carried over the same distance. The reader should note that the term "load factor" is also used in Chapter 8 in the context of the loads from flight maneuvers on the airplane structure.
The *transport productivity* is defined as the transport product delivered per unit

time. Usually, it is based on block time, so that we can write

$$P_h = \frac{W_p R}{E_B} = W_p V_B. \tag{15.50}$$

From the combination of the payload range diagram in Figure 15.15a and the block speed-range relationship in Figure 15.15c, we obtain the curve of Figure 15.15d. This shows the typical variation of transport productivity with block distance. From the diagrams, we see that the maximum value of P_h occurs at the design range.

The *revenue-earning capacity* is the transport product per year. If U is the annual flight utilization of the airplane in hours, we have

$$P_y = P_h U = W_p V_B U. \tag{15.51}$$

For long-haul routes, the annual utilization might reach a value of 4500 hours. This figure gradually decreases as the route distance (block time) becomes shorter (Reference 52).

Besides the preceding parameters, the specific costs of operating a commercial airplane, that is to say, the costs per tonkm or costs per hour are as much of importance to its economic value.

The operating costs are usually broken down into direct and indirect costs. The direct operating costs (DOC) are those which are associated with flying operations. These may include maintenance, crew, airplane service, landing fees, depreciation of capital invested, insurance, and fuel. The indirect operating costs (IOC), on the other hand, are independent of the characteristics of the airplane since they are connected with the costs of operating an airline. They may encompass management, administration, sales, housing, and depreciation of ground properties and equipment.

Beyond doubt, when considering the economic value of an airplane we shall specially be interested in the direct operating costs as a criterion.

As a means of estimating DOC for comparative purposes, standard methods were published by the Society of British Aircraft Constructors (SBAC) in 1959 and by the Air Transport Association of America (ATA) in 1967 (References 52 and 53). To reflect the effects of inflation and changing technology, renewed and updated cost models have been developed recently. See, for example, References 54 and 55. These sources will aid the reader in obtaining a proper understanding of the airplane related cost problem.