

Analysis and Modelling of Physical Transport Phenomena

The cover pictures represent the instantaneous temperature fields in the proximity of hot and cold walls (inside thermal boundary layers) in Rayleigh-Bénard convection of air at $Ra = 10^9$, large-eddy simulation. S. Kenjereš, 2006.

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Preface

These lecture notes contain the course material Advanced Physical Transport Phenomena, offered in the Master's programme in Applied Physics at Delft University of Technology. The notes follow in part the concept and content of the book *Fysische Transportverschijnselen II* (in Dutch) by Hoogendoorn and Van der Meer (Delft University Press, 1991). However, a significant amount of new material on turbulent flows, convective processes and numerical methods has now been included. The course aims at providing graduate students with an overview of analytical, numerical and modelling methods for solving problems of heat and fluid flow, following a unified and comparative approach.

The course is divided into four parts. The first part gives the conservation laws for mass, momentum and energy in general differential forms, accompanied with the relevant constitutive relations, physical and mathematical classification of equations, and their boundary conditions. This concise introduction is just a generalisation of the macroscopic conservation laws considered in basic courses on Physical Transport Phenomena at the bachelor's level.

Part II covers a number of classical analytical methods for solving some generic problems in heat, mass and momentum transfer. In addition to providing insight into the basic physics of transport phenomena, this part is meant to encourage students to master the analytical tools and to use analytical approaches for gaining a physical intuition by solving elementary problems in idealized situations. It also illustrates the limitations and constraints of analytical methods in solving complex problems in transport phenomena.

Part III introduces numerical methods for computer-aided solutions of complex problems that are not tractable by analytical approaches. It is, in fact, an introduction to computational fluid dynamics (CFD) and computational heat and mass transfer (CHMT), which have recently emerged as major tools for solving heat and fluid flow problems in engineering and environmental applications. With that in mind, the focus is on the finite-volume discretization of the conservation laws, which is the main approach in industrial CFD and CHMT. In addition to introducing basic concepts of equation discretization and their numerical solution, this chapter aims at illustrating the potential but also the limitations and inherent snares of computational methods. Rather than providing a full coverage of various schemes and solution methods, the chapter aims at developing a critical attitude among students and an ability to recognize potential errors and numerical contaminations.

Part IV deals with turbulent convection, considered to be the most widespread mode of transport in real life problems, but also the most challenging both for analytical and

computational treatment. After introducing some basic notions on turbulence relevant to its modelling, we discuss the features and phenomena of a series of generic wall-bounded flows. This should provide students with basic physical insight based solely on similarity and scaling arguments. Limitations of detailed numerical simulation are then discussed, together with the need for mathematical modelling of turbulence and associated turbulent transport processes of heat and mass. The last section in this part covers the basics of turbulence modelling, its scope and limitations. The practice and the rationale of turbulence modelling are illustrated by detailed derivation of the $k - \varepsilon$ model and its closure. This is accompanied by physical interpretations wherever possible, which provide an insight into modelling arguments, levels of approximation and model limitations. The section is closed with a brief overview of other two-equation eddy-viscosity/diffusivity models and an introduction into non-linear eddy-viscosity- and second-moment closures. The lecture notes contain a number of worked-out examples, especially in Part II (analytical methods).

Authors, December 2007, Delft, The Netherlands