16 Signal generation

Measurement signals are almost invariably aperiodic but pure periodic signals are also important in instrumentation. More often than not, they serve as auxiliary signals like, for instance, when functioning as modulated signal carriers (see in this connection Chapter 17) or when used as test signals, for example to analyze a system's frequency transfer function (see for further details Chapter 21). It is therefore worth knowing how periodic signals are generated. An instrument that produces a periodic signal is called a signal generator. If the signals produced are just sinusoidal then such an instrument is known as an oscillator. Indeed, the principle of precisely how oscillators work will be explained in the first part of this chapter. Finally, there are instruments that are able to generate other periodic signals. Function generators, to name but one sort, produce divergent periodic signals such as: square wave, triangular, ramp and sine-shaped signals. All these kinds of instruments will be described in the second part of this chapter.

16.1 Sine wave oscillators

There are various ways to generate a sinusoidal signal, one way is by solving a second order differential equation using analog electronic circuits, a principle that is outlined in this section. A second way involves starting with a symmetric, rectangular or triangular signal. The sine shape is obtained either by filtering out the superharmonics and keeping the fundamental, or by reshaping the signal using resistance-diode networks of the type described in Chapter 14. Obviously, the accuracy of this last method will depend very much on the quality of such non-linear converters. A third way is by synthesizing arbitrary periodic signals with the aid of a computer. There the processor generates a series of successive codes that are converted into an analog signal by a DA converter (Chapter 18).

16.1.1 Harmonic oscillators

The general solution to the linear differential equation

$$a_0 \frac{d^2 x}{dt^2} + a_1 \frac{dx}{dt} + a_2 x = 0 \tag{16.1}$$

is

$$x(t) = \hat{x} e^{-\alpha t} \sin(\omega t + \varphi) \tag{16.2}$$

in which $\alpha = a_1/2a_0$, $\omega = \sqrt{(a_2/a_0 - a_1^2/4a_0^2)}$ and $\hat{x}$ and φ are arbitrary constants. In accordance with the sign α, $x(t)$ remains a sinusoidal signal with an exponentially decreasing amplitude ($\alpha > 0$) or an exponentially increasing amplitude ($\alpha < 0$). Only when $\alpha = 0$, $x(t)$ is a pure sine wave with constant amplitude. Consequently, the coefficient α is termed the damping factor. It is fairly easy to design an electronic circuit with voltages and currents to satisfy equation (16.1). In order to generate pure sine waves at a constant amplitude the coefficient a_1 should be kept at zero which does require extra effort.
The signal derivatives and integrals are obtained from inductances and capacitances. Active elements are required to keep the damping factor at zero. In addition to this, some kind of feedback appears to be necessary. An example will be given to illustrate the basic principle (in practical terms the example is of little significance but it is useful for explanatory purposes). Figure 16.1 provides a block diagram of an electronic system with two differentiators and one amplifier in series.
The output is connected straight to the input which means that:

$$v_o = K\tau^2 \frac{d^2 v_o}{dt^2} \tag{16.3}$$

The solution to this linear, homogeneous differential equation is $v_o = \hat{v}\sin(\omega t + \varphi)$, with $\omega^2 = -1/K\tau^2$. Evidently K must be negative (it is an inverting amplifier). For $K = -1$ the frequency of the signal produced is $f = 1/2\pi\tau$. Instead of using differentiators one can alternatively take integrators. As was seen in Chapter 13, an integrator is more stable and produces less noise than a differentiator. The differential equation, however, remains the same.

The amplitude can have any value between the system's signal limits, it is not fixed by circuit parameters. As long as a_1 in Equation (16.1) is zero the amplitude, once present, will remain constant. In an actual circuit, though, the component values will vary perpetually, for instance because of temperature fluctuations, so the condition $a_1 = 0$ will not be met for a long time. This means that somehow the system has to be controlled if the amplitude is to be fixed at a prescribed value. There is also another reason why such a control circuit is needed. When the system is switched on the amplitude is zero. As long as the damping factor α is zero, or positive, the amplitude will remain at zero. Therefore α must be negative for a short period so that the amplitude can be allowed to rise to the desired value. When that value has been reached α must revert again to zero.

In the circuit given in Figure 16.1 the term a_1 is obtained by simply adding a fraction β of v_2 to the voltage v_3. The output voltage thus becomes:

$$v_o = K\left(\tau^2 \frac{d^2 v_o}{dt^2} + \beta\tau \frac{dv_o}{dt}\right) \tag{16.4}$$

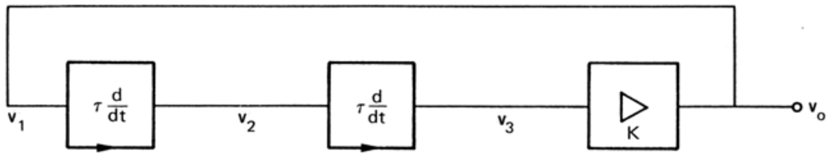


Figure 16.1. An oscillator with two differentiators and an amplifier.

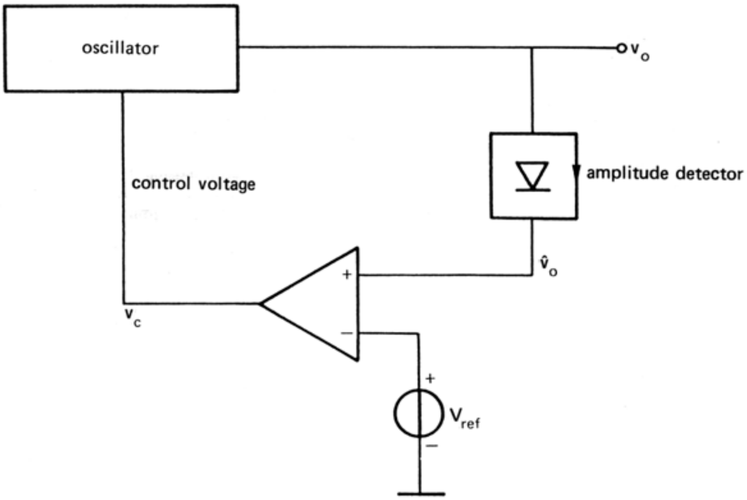


Figure 16.2. The principle of an oscillator with amplitude control.

Whether the output amplitude increases or decreases depends on the fraction β. With $\beta = 0$ a sine wave with steady output is generated.

Figure 16.2 shows how an oscillator with electronic amplitude control is set up. The control system consists of the following components:

- the amplitude detector. Its output $\hat{v}_o$ is a measure of the amplitude of the generated sine wave (like, for instance, the peak detector shown in Section 9.2.2 or a rectifier with low-pass filter);
- a reference voltage V_{ref};
- a control amplifier to amplify the difference between the reference voltage V_{ref} and the peak value $\hat{v}_o$;
- a control element in the oscillator, this can be an electronically controllable resistance (JFET, thermistor; photoresistor) or an analogue multiplier. This control element affects the factor a_1 in Equation (16.1) or β in Equation (16.4) and therefore also the damping factor α.

As the signal power is determined by the square of the signal voltage amplitude or the current amplitude a control element based on heat dissipation may be utilized. An element that is widely used for this purpose is the thermistor which is part of the oscillator network that operates in such a way that at increasing amplitude (i.e. at decreasing resistance) further increase is halted. This method, requiring merely a single component, is extremely simple but control is slow due to the thermal nature of the network. What should also be pointed out is that in a steady state the amplitude depends on the thermistor parameters as well as on the heat resistance to the environment (i.e. the environmental temperature). This method is not suited to high amplitude stability.

16.1.2 Harmonic oscillator circuits

The relationships between the voltages and the currents in an electronic network are given as linear differential equations. We have introduced complex variables (see Chapter 4), mainly because the solving of these equations inevitably becomes rather time-consuming. In its steady state, a harmonic oscillator generates a pure sine wave which means that oscillators of this kind can be analyzed with the help of complex variables. Equation (16.3), for example, which belongs to the circuit given in Figure 16.1, can be written as $V_o = K\tau^2(j\omega)^2 V_o$ which therefore means that $\omega^2 = -1/K\tau^2$.
Any harmonic oscillator is composed of at least one amplifier and one passive network with a frequency-selective transfer. In most cases, an oscillator can be modeled in the way shown in Figure 16.3. When mutual loading can be ignored or when this effect can be discounted by A or β then $V_o = AV_i$ and $V_i = \beta(\omega)V_o$ so that $A\beta(\omega) = 1$. This complex equation is termed the oscillation condition. The conditions for oscillation and oscillation frequency are provided when this equation is solved. Several examples will now be given to illustrate this point.

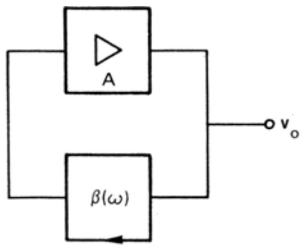


Figure 16.3. A basic harmonic oscillator diagram showing how the amplifier output is fed back to the input via a network by means of frequency selective transfer.

The Wien oscillator

With a Wien oscillator, the feedback comes from two resistors and two capacitors arranged as voltage dividers with band-pass characteristics (Figure 16.4). Under the conditions $R_1 = R_2 = R$ and $C_1 = C_2 = C$, the transfer function of this Wien network is

$$\beta(\omega) = \frac{V_i}{V_o} = \frac{1}{3 + j\omega\tau + 1/j\omega\tau} \tag{16.5}$$

The oscillation condition is $A\beta(\omega) = 1$, hence:
$3 + j\omega\tau + 1/j\omega\tau = A$
The real parts on the left and right-hand sides of this equation must be equal, just like the imaginary parts. This will result in two equations:

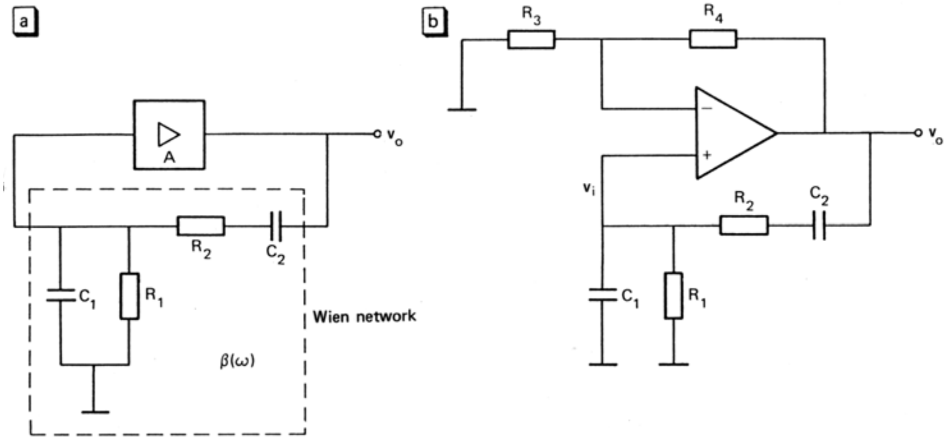


Figure 16.4. (a) An oscillator created according to the principle given in Figure 16.3 with a Wien network, (b) a Wien oscillator that has one operational amplifier.

$$A = 3 \tag{16.6}$$

$$\omega = \frac{1}{\tau} \tag{16.7}$$

Equation (16.6) describes the condition for constant amplitude. If $A > 3$ the amplitude will increase while with $A < 3$ it will decrease. Equation (16.7) gives the frequency of the signal generated. Figure 16.4b shows a possible Wien oscillator configuration without amplitude control. The calculation of the oscillation conditions arising directly from this circuit results in the equations $\omega = 1/\tau$ and $R_4 = 2R_3$. This is not a surprising result, without the Wien network the amplifier acts as a non-inverting amplifier with a gain of $1 + R_4/R_3 = 3$ for the condition previously found.

The phase-shift oscillator

The basic idea underlying the phase-shift oscillator is depicted in Figure 16.5. Here the feedback network consists of three cascaded low-pass RC filters. If the values of the resistors and capacitors are such that the time constants are equal but the sections do not load each other (cf. Figure 8.13) then the transfer will be $\beta(\omega) = 1/(1 + j\omega\tau)^3$. The oscillation condition is simply $K = (1 + j\omega\tau)^3$. If this equation is divided into real and imaginary parts then this will result in $K = -8$ and $\omega^2\tau^2 = 3$.

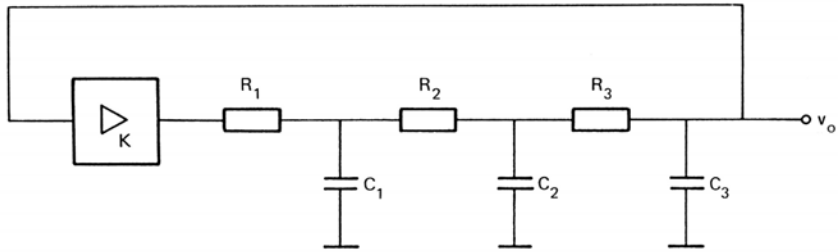


Figure 16.5. A phase-shift oscillator with three low-pass RC-sections.

The two-integrator oscillator

The last example of a harmonic oscillator to be given is the two-integrator oscillator or the dual integrator loop (Figure 16.6). The oscillator condition can easily be established: $(-1/j\omega\tau)^2(-1) = 1$, hence $\omega = 1/\tau$. The oscillation frequency can be varied by simultaneous changing the resistors R_1 and R_2 or the capacitors C_1 and C_2. The same effect can be achieved by having two adjustable voltage dividers (potentiometers) situated at the integrator inputs (Figure 16.7). If k is the attenuation of the voltage dividers then the oscillation condition will be $(1/j\omega\tau)^2k^2(-1) = 1$ so that $\omega = k/\tau$. The oscillation frequency is proportional to the attenuation of the potentiometers.

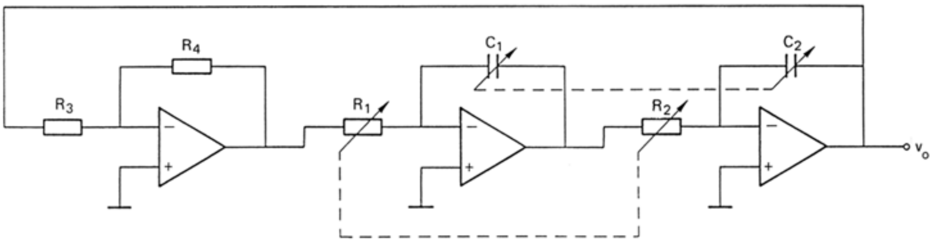


Figure 16.6. A two-integrator oscillator consisting of an amplifier and two integrators. The frequency is adjustable with both the capacitors C and the resistors R. Usually C_1 and C_2 are switched in stages by a factor of 10 (rough frequency adjustment), whereas R_1 and R_2 remain the potentiometers for the fine tuning of the frequency.

In Figure 16.7 an amplitude stabilization circuit is also given. A fraction β of v_2 is added via R_5 to the leftmost inversion (cf. Equation 16.4). The value of β is multiplied by the output of the control amplifier where the inputs are the rectified oscillator output voltage and a reference voltage. With the control circuit the difference between V_{ref} and $\hat{v}_o$ tends to be zero.

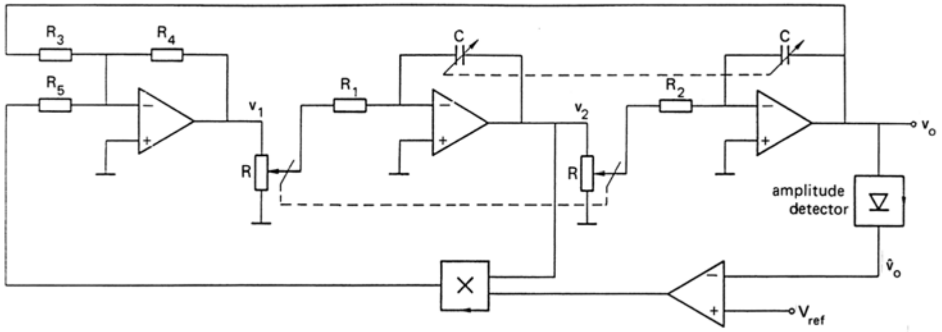


Figure 16.7. A possible configuration for the two-integrator oscillator with an amplitude stabilization circuit.

16.2 Voltage generators

Periodic non-sinusoidal signals are frequently used in instrumentation systems. In combination with an oscilloscope (see Chapter 21) it is possible to visualize a system's step and pulse responses or to measure its rise and delay time. This is done by connecting a periodic square wave or pulse signal to the test system input and observing its output on an oscilloscope or a computer monitor. Triangular or ramp signals make it possible to determine a system's non-linearity. They can also function as control signals in various actuators and be used for the testing of systems or products. Pulse and square wave signals are widely used in digital systems, for instance for synchronization.

In this section we shall discuss a number of the generators used with non-sinusoidal periodic voltages. Most of these instruments are based on the periodic charging and discharging of a capacitor.

16.2.1 Triangle voltage generators

A triangle generator periodically charges and discharges a capacitor with a constant current (Figure 16.8). At constant current, the voltage across a capacitance constitutes a linear time function. This voltage is connected to a Schmitt-trigger (14.2.2) with output levels V^+ and V^- (Figure 16.9). The control circuit controls the switches in such a way that for $v_s = V^+$ the capacitor is charged with current I_1 and for $v_s = V^-$ it is discharged with current I_2. This results in a triangular voltage across C. Any time v_o passes the Schmitt-trigger switching levels the two states will be automatically interchanged. The peak values of the triangle are equal to the switching levels of the Schmitt-trigger. The slope of the triangle can be adjusted by varying the values of currents I_1 and I_2.

Figure 16.10 shows a simple configuration with one integrator, a Schmitt-trigger and an inverting amplifier. At equal charging and discharging currents and $V^+ = V^-$, a symmetrical triangle voltage is obtained. The circuit simultaneously generates a square wave voltage.

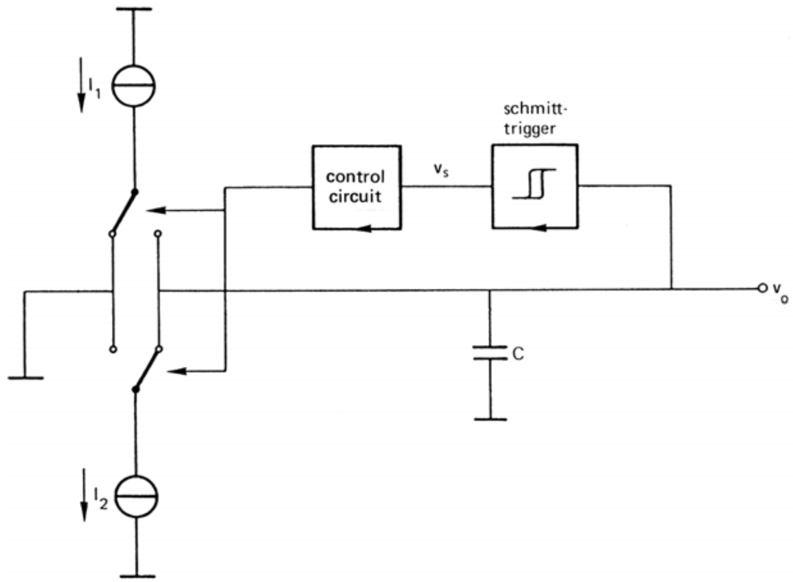


Figure 16.8. A functional diagram of a triangle generator. The triangular voltage is produced by periodically charging and discharging a capacitor with a constant current.

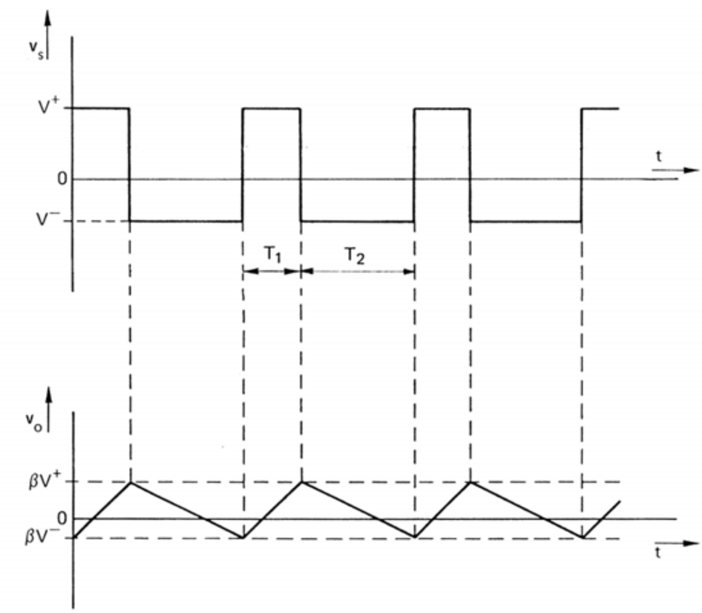


Figure 16.9. The voltages seen in Figure 16.8. V^+ and V^- are the positive and negative power supply voltages, βV^+ and βV^- are the Schmitt-trigger switching levels. The rise and fall time of v_o is determined by I_1, I_2 and C.

The ratio between the time T_1 and the total period time $T = T_1 + T_2$ of such periodic signals is the signal's duty cycle. A symmetric signal has a duty cycle equivalent to 50%. The duty cycle can be changed by varying one of the currents in Figure 16.8.

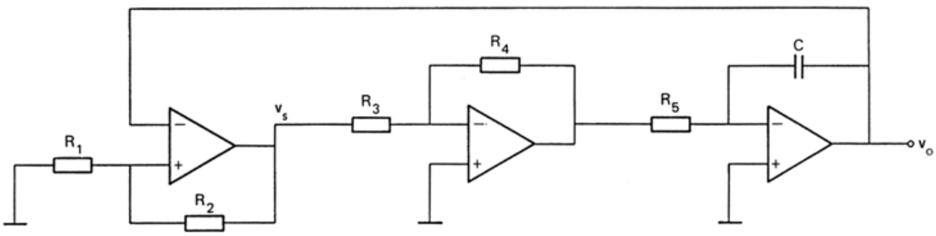


Figure 16.10. A simple configuration for a triangle generator using a Schmitt-trigger, an inverting amplifier and an integrator.

16.2.2 The ramp generator

A ramp voltage can be viewed as a triangular voltage with one vertical slope. Such a short fall time is obtained by discharging a charged capacitor over a switch. The switch is controlled by a Schmitt-trigger (Figure 16.11). For $v_s = V^+$ the switch is on and for $v_s = V^-$ it is off. The capacitor is part of an integrator that has an input connected to a fixed reference voltage V_{ref1}. The output of the integrator rises linearly over the course of time until the switch goes on and the capacitor discharges. In that way, the output reverts to zero. The process will start all over again when the switch is released.

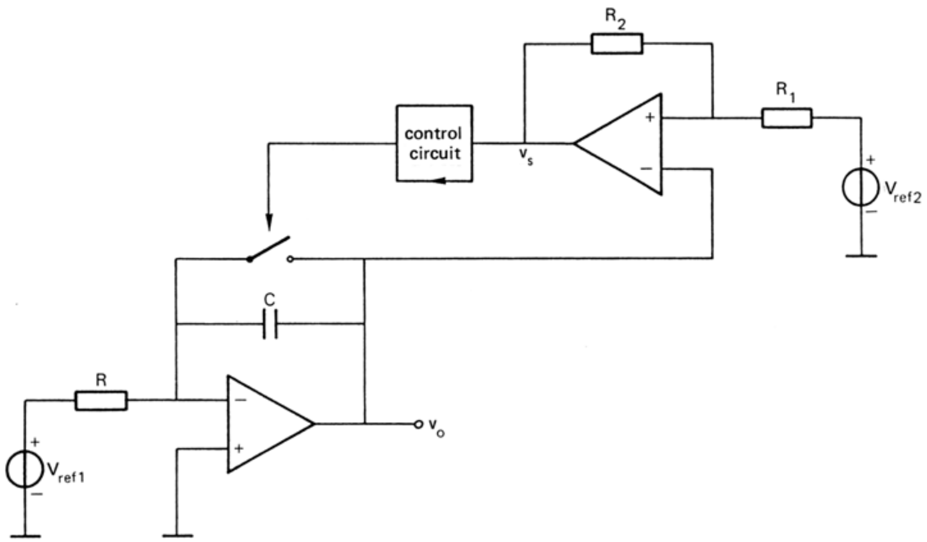


Figure 16.11. A functional diagram for a saw-tooth generator. The output voltage is produced by charging a capacitor with a constant current and discharging it through a switch.

Figure 16.12 shows the various output voltages in the case of a negative reference voltage. The switch goes on as soon as v_o reaches the Schmitt-trigger's upper switching level. The capacitor discharges very rapidly, the output voltage then drops to zero and remains at zero as long as the switch is on. This means that the lower switching level of the Schmitt-trigger (which is βV^- for $V_{ref2} = 0$, see Section 14.2.2) must be higher than zero otherwise the Schmitt-trigger will remain in the $v_s = V^-$ state, the switch will never go off again and the output will stay at zero. This explains the need for the second reference voltage V_{ref2}. The switching levels of the Schmitt-trigger are determined by R_1, R_2 and V_{ref2}.

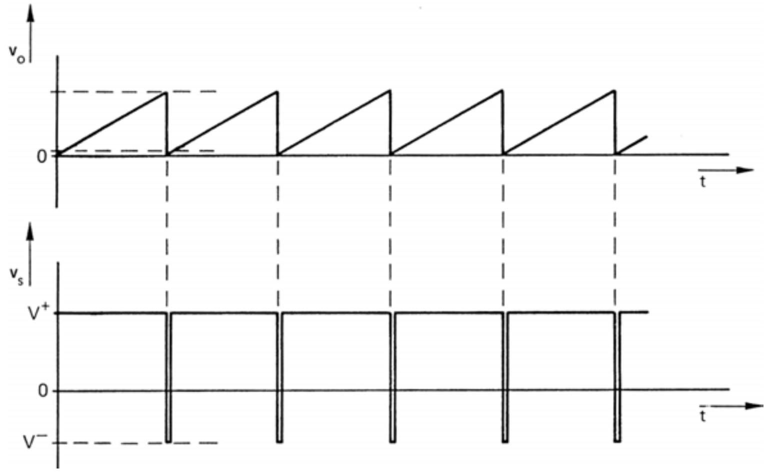


Figure 16.12. The output voltage v_o of the integrator and the output voltage v_s of the Schmitt-trigger in the circuit seen in Figure 16.11. The dotted lines in v_o represent the switching levels of the Schmitt-trigger. For $v_s = V^+$ the switch is on, for $v_s = V^-$ it is off.

Example 16.1

The power supply voltages of the operational amplifiers in Figure 16.11 are $V^+ = 15$ V and $V^- = -15$ V. The output levels of the Schmitt-trigger are assumed to be equal to the supply voltages or, in other words, +15 and –15 V. The switching levels are calculated as follows. Both levels are described as:

$$V_{ref2}\frac{R_2}{R_1+R_2}+v_s\frac{R_1}{R_1+R_2}$$

The upper level is for $v_s = +15$ V, so it amounts to:

$$V_{ref2}\frac{R_2}{R_1+R_2}+15\frac{R_1}{R_1+R_2}$$

which is also the upper ramp peak v_o. The lower switching level is for $v_s = -15$ V, which means that it is:

$$V_{ref2}\frac{R_2}{R_1+R_2}-15\frac{R_1}{R_1+R_2}$$

The minimum value of v_o is zero (the discharged capacitor). To allow the ramp to run from +1 V to +10 V, the upper level must satisfy:

$$V_{ref2}\frac{R_2}{R_1+R_2}+15\frac{R_1}{R_1+R_2}=10$$

and the lower level must be:

$$V_{ref2}\frac{R_2}{R_1+R_2}-15\frac{R_1}{R_1+R_2}=1$$

These equations establish the ratio between R_1 and R_2 as well as V_{ref2}: 3/7 and +7.9 V, respectively.

The width of the pulse-shaped voltage v_s is determined by the discharge time of the capacitor and the delay times of the switch and the Schmitt-trigger. This pulse is, in any case, very narrow thus creating the extremely steep rising edge of v_o. The Schmitt-trigger could be replaced by a comparator (a Schmitt-trigger without hysteresis). However, due to the on-resistance of the switch (Section 15.1), the discharge period is not zero (it is actually an exponentially decaying curve). Due to the Schmitt-trigger hysteresis, the switch will only go off again if v_o is sufficiently close to zero, irrespective of the discharge time.
The frequency of the generated ramp voltage is determined by the time constant RC, by V_{ref1} and by the hysteresis of the Schmitt-trigger. The control circuit adjusts the output of the Schmitt-trigger to the levels appropriate for activating the switches.

16.2.3 Square wave and pulse generators

Most circuits used for the generation of square wave and pulse-shaped signals are composed of resistors, capacitors and a number of digital circuits, they can also be created with an operational amplifier. The principle of operation is essentially the same. Figure 16.13a shows a square wave generator with a Schmitt-trigger. The Schmitt-trigger output (the operational amplifier together with R_1 and R_2) is connected to the input via an integrating RC-network. Again, the capacitor is periodically charged and discharged but this time that is not done with a constant current but via the resistor R instead. The voltage v_c across the capacitor therefore approaches the respective levels V^+ and V^- exponentially. The switching levels are βV^+ and βV^-. The frequency of the generated signal can be derived from Figure 16.13b. It appears that the period time is

$$T=\tau\ln\left(\frac{V^- -\beta V^+}{V^- -\beta V^-}\cdot\frac{V^+ -\beta V^-}{V^+ -\beta V^+}\right) \tag{16.8}$$

with $\tau = RC$.

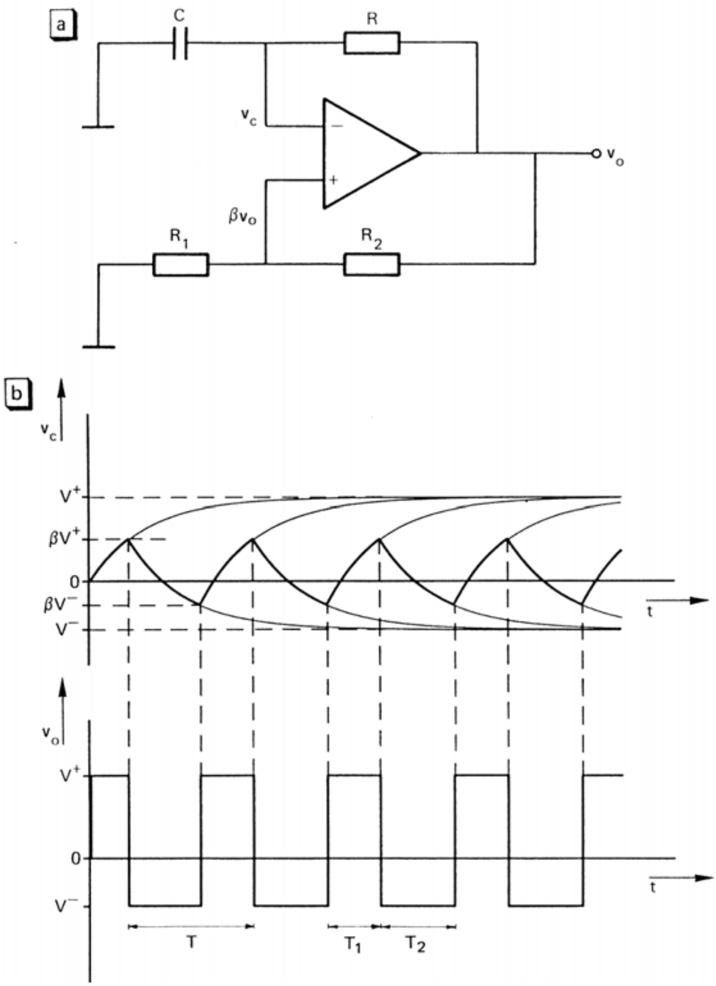


Figure 16.13. (a) The circuit and (b) the corresponding voltages of a square-wave generator constructed with a Schmitt-trigger.

16.2.4 Voltage-controlled oscillators

Many applications require a generator with a frequency that can be controlled by a voltage. Such generators are called voltage-controlled oscillators (VCO) or sweep generators. The latter name is an allusion to the chance to linearly or logarithmically change the frequency of the VCO. The control voltage is a triangular or ramp voltage that sweeps the frequency of the VCO between two adjustable values.

The oscillators of Section 16.1 are not suitable for a VCO because their frequency is determined by resistances and capacitances which can barely be electronically controlled, at least not over a very wide range. The generators with periodically charging and discharging capacitors are better suited to VCOs. The charging time and therefore also the frequency is mainly determined by the charge current. For instance,

the frequency of the ramp generator seen in Figure 16.11 is directly proportional to the reference voltage V_{ref1}.
The principle of the VCO is the same as that of the triangle generator seen in Figure 16.8. The frequency of the triangular voltage is proportional to the charge current I_1 and the discharge current I_2. A VCO may be obtained by employing a voltage-to-current converter to produce these currents.
VCOs are available as integrated circuits. Figure 16.14 shows a block diagram with the connections for such circuits. The circuit in question contains two buffered outputs: one for a triangular voltage, the other for a square wave signal. The frequency is controlled by the input voltage v_i. Depending on the type, the sweep range will be a factor of 3 to 10. The sweep range can be changed by external resistances or capacitances within a 1 Hz to 1 MHz range.

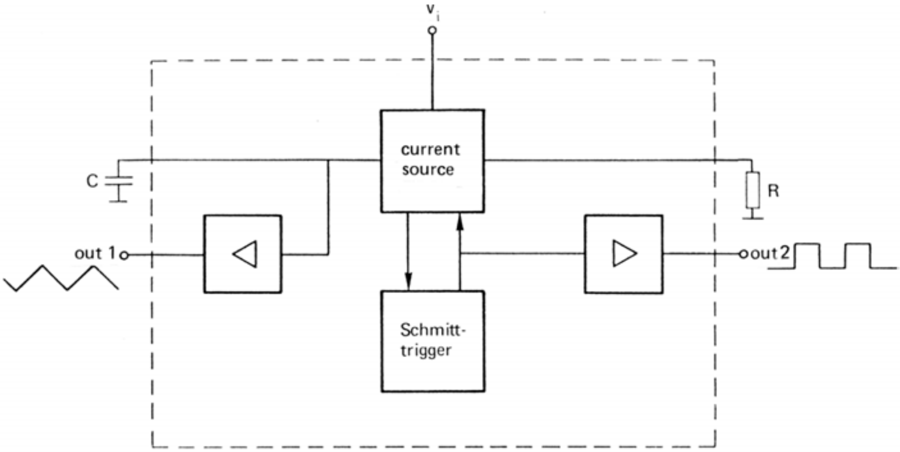


Figure 16.14. An integrated voltage-controlled oscillator (VCO) with triangular and square wave outputs.

There are also VCO types with a much wider sweep range, their output is usually just a pulse or a square wave. The frequency sweep of such kinds of VCOs may be more than a factor of 1000 while the frequency range rises to over 10 MHz.
Such circuits are usually called voltage-to-frequency converters, in particular when there is an accurate relationship between the control (or input) voltage and the output frequency. They are used to convert sensor signals into frequency-analog signals in order to improve noise immunity when transmitting sensor signals over large distances.

SUMMARY

Sine wave oscillators

- One way of generating a sine wave is by using the electronic solution to a second order differential equation which is a sine wave. Other methods involve: filtering the harmonics from a triangular or rectangular signal wave, sine-shaping by means of non-linear transfer circuits and synthesizing with a computer.

- A harmonic oscillator consists of an amplifier, a frequency-selective network and an amplitude control circuit. The parameter that has to be controlled is the damping factor of the second order system.
- The oscillation condition is $A\beta(\omega) = 1$, in which A and β are the amplifier transfers and the feedback network. The solving of this equation creates the conditions for the gain and oscillation frequency.
- The most frequently used types of oscillators are: the Wien oscillator (an amplifier with a Wien network), the phase-shift oscillator (an amplifier with a cascaded series of RC networks) and the two-integrator oscillator or dual integrator loop (comprising a loop of two integrators and an inverting amplifier).

Voltage generators

- A triangular or ramp voltage can be produced by alternatingly charging and discharging a capacitor with a constant current. The switching moments are determined by the Schmitt-trigger switching levels and the peak-to-peak value by its hysteresis interval.
- The duty cycle of a pulse-shaped signal is the ratio between the "on" time and the period time.
- A voltage-controlled oscillator (VCO) is a generator with a frequency that can be varied according to the voltage. The sensitivity of a VCO is given in Hz/V or kHz/V.

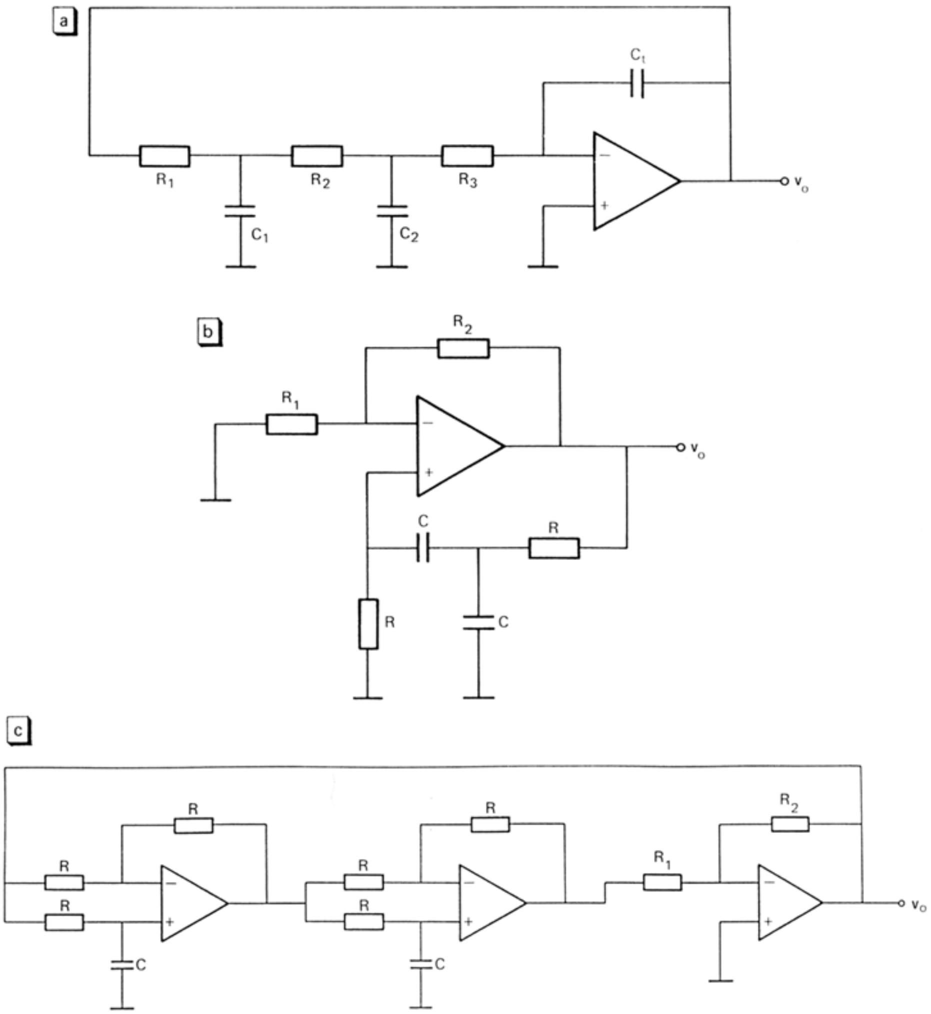
EXERCISES

Sine wave oscillators

16.1 Why does a harmonic oscillator need an amplitude control circuit?

16.2 In $\alpha < 0$ the solution to Equation (16.2) is a sine wave with exponentially increasing amplitude. However, when the system is switched on, the initial amplitude will be zero. Explain why the circuit will start to oscillate?

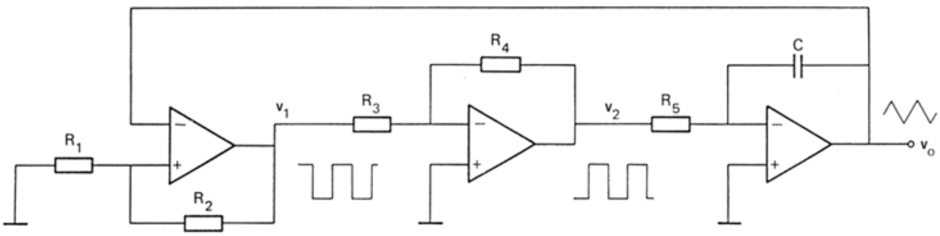
16.3 Find the oscillation conditions for circuits a-c given below. Determine the oscillation frequency and the conditions for the component values. The operational amplifiers may be considered to be ideal. To simplify the calculations, take $R_1 = R_2 = R$; and $C_1 = C_2 = C$.



16.4 Look at the sine-wave oscillator given in Figure 16.4b. The amplitude control is realized by replacing R_4 with a thermistor. The following applied component properties are given: $R_3 = 500\ \Omega$; $R_4 = R_{th} = 10^4 e^{5200(1/T - 1/T_0)}$. The heat resistance between the thermistor and the ambient is 100 K/W. T_0 = 273 K, the ambient temperature is 300 K. Find the amplitude of the output signal.

Voltage generators

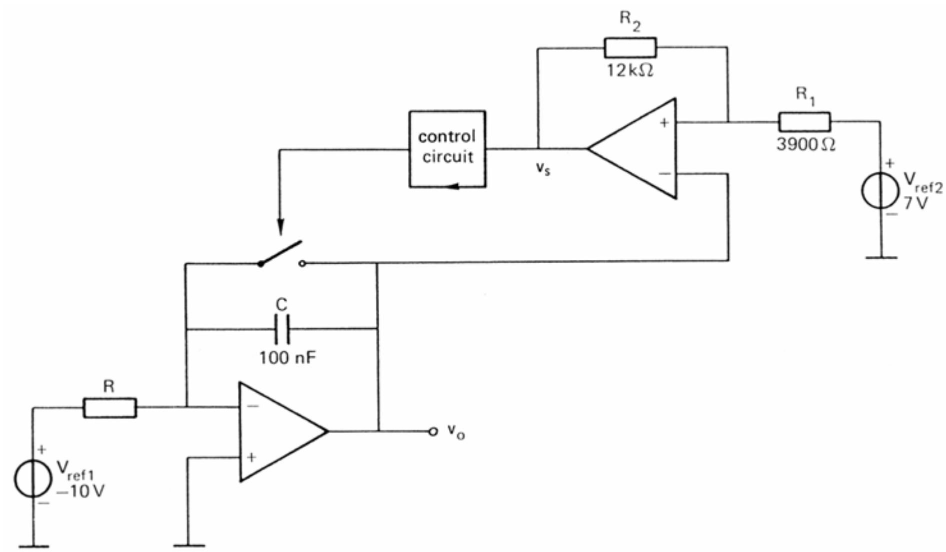
16.5 Imagine that in the case of the triangle generator given below the following conditions hold: $R_3 = 5R_4$, $R_1/(R_1 + R_2) = \beta$ and $R_5C = \tau$. Discuss the changes in v_o when the value of R_1 increases.



16.6 Calculate the duty cycle of the voltages v_1 and v_2 in the Figure accompanying Exercise 16.5, in the case of $V^+ = 15$ V and for $V^- = -5$ V.

16.7 Calculate the frequency, average value and amplitude of the triangular voltage v_o in the circuit given in Exercise 16.5 when $V^+ = 15$ V, $V^- = -5$ V, $R_5 = 10$ kΩ, $C = 100$ nF and $\beta = 0.5$.

16.8 Examine the ramp generator depicted below. The output frequency of v_o is 1 kHz. Establish the value of R. All components may be considered ideal and the output levels of the Schmitt-trigger are $v_s = V^+ = 18$ V or $v_s = V^- = -12$ V.



16.9 Study the generator in the preceding exercise. By adding only one voltage source the average output voltage can be adjusted to zero. How can it be adjusted and what is the value?

16.10 Look again at the ramp generator in Exercise 16.8. Find the amplitude and the frequency of the output signal v_o for $R = 100$ kΩ, the switch's delay time is 2 ms but apart from that the switch is ideal.

16.11 Modify the circuit in Exercise 16.8 so that the output has a negative slope.

16.12 Determine the duty cycle of the output signal in Figure 16.13 as a function of V^+, V^-, β and $\tau = RC$. Find the condition for a 50% duty cycle.