Electromagnetic Waves

— A Repetitive Guide

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Preface

A course on Electromagnetic Waves almost inevitably confronts the student with a vast amount of formulas. Obviously, it is not expected from the student to learn all these by heart. A common way out of this dilemma is the compilation of a table of formulas. Drawbacks are that this can hardly be made to contain enough items and, more seriously, that it does not show how the items are applied and what their underlying relations are. It is precisely for these aspects for which the student has to develop an understanding. This *Repetitive Guide* is meant to help students in obtaining the right attitude towards the material presented in the main course book entitled *Electromagnetic Waves* by H. Blok and P.M. van den Berg. In essence, this guide contains an outline of that book. The basic formulas are listed within their context, and the main types of derivations are shown, most times by taking one specific case as an example. In this way, the student is offered a framework that can serve as a guideline for deriving formulas that apply in corresponding situations. He or she who has successfully completed a course on Electromagnetic Waves, has learned to tackle a problem using this guide instead of searching for the relevant formula in the main course book. This reflects the way of working in practical science: One rarely has available the formula that is needed in a specific case, but one will as much as possible follow the lines set out by others in deriving it. In this respect this *Repetitive Guide* can help develop a skill that is particularly important for students at a University of Technology.

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Preface to the second edition

This edition is idential to the first edition, except that a number of errors have been corrected.

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4. Two-dimensional Electromagnetic Waves

Plane waves:

A plane wave propagating in the positive x_1- and x_3-direction is written as

$$\begin{aligned} \hat{\boldsymbol{E}} &= \hat{\boldsymbol{e}}(s)\exp(-\gamma_1 x_1 - \gamma_3 x_3)\,, \\ \hat{\boldsymbol{H}} &= \hat{\boldsymbol{h}}(s)\exp(-\gamma_1 x_1 - \gamma_3 x_3)\,, \end{aligned} \quad \Bigg| \quad \boldsymbol{\gamma}\cdot\boldsymbol{\gamma} = \gamma_1^2 + \gamma_3^2 = (\sigma + s\varepsilon)s\mu\,.$$

In case of *steady-state*, the complex propagation vector $\boldsymbol{\gamma}$ is written as

$$\boldsymbol{\gamma}(j\omega) = \boldsymbol{\alpha}(\omega) + j\boldsymbol{\beta}(\omega)\,,$$

where $\boldsymbol{\alpha} = \{\alpha_1, 0, \alpha_3\}$ is the *attenuation vector* and $\boldsymbol{\beta} = \{\beta_1, 0, \beta_3\}$ is the *phase vector*. For *uniform* plane waves $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ have the same direction.

Uniform plane waves:

Let $\boldsymbol{s} = s_1\boldsymbol{i}_1 + s_3\boldsymbol{i}_3$ be a unit vector, then a uniform plane wave propagating in the $\boldsymbol{s}$-direction is written as

$$\begin{aligned} \hat{\boldsymbol{E}} &= \hat{\boldsymbol{e}}(s)\exp[-\gamma(s_1 x_1 + s_3 x_3)]\,, \\ \hat{\boldsymbol{H}} &= \hat{\boldsymbol{h}}(s)\exp[-\gamma(s_1 x_1 + s_3 x_3)]\,, \end{aligned} \quad \Bigg| \quad \gamma^2 = (\sigma + s\varepsilon)s\mu\,.$$

The electric-field vector $\hat{\boldsymbol{e}}$, the magnetic-field vector $\hat{\boldsymbol{h}}$ and the propagation-direction vector $\boldsymbol{s}$ form a mutually perpendicular and right-hand triad.

In case of *steady-state*, the complex propagation coefficient is obtained as

$$\gamma = \alpha + j\beta = [(\sigma + j\omega\varepsilon)j\omega\mu]^{\frac{1}{2}},\ \mathrm{Re}[\ \ ]^{\frac{1}{2}} \geq 0\,,$$

where α is the attenuation coefficient and β is the phase coefficient, while the wavelength follows from $\lambda = 2\pi/\beta$. The time average Poynting's vector is given by

$$\langle\boldsymbol{S}\rangle_T = \tfrac{1}{2}\mathrm{Re}\left[\hat{\boldsymbol{E}} \times \hat{\boldsymbol{H}}^*\right] = \boldsymbol{S}_0\exp[-2\alpha(s_1 x_1 + s_3 x_3)]\,.$$

Parallelly polarized waves: $\hat{e}_1 \neq 0$, $\hat{e}_3 \neq 0$ and $\hat{h}_2 \neq 0$ (→ Fig. 8.2).
The electric field strengths follow from the fundamental unknown $\hat{h}_2$ as

$$\hat{e}_1 = \frac{\gamma_3}{\sigma + s\varepsilon}\,\hat{h}_2\,, \quad \hat{e}_3 = \frac{-\gamma_1}{\sigma + s\varepsilon}\,\hat{h}_2\,,$$

while for steady-state uniform planes the energy transfer follows from

$$\boldsymbol{S}_0 = \tfrac{1}{2}\mathrm{Re}\left[\hat{\boldsymbol{e}} \times \hat{\boldsymbol{h}}^*\right] = \tfrac{1}{2}\mathrm{Re}\left[Z(j\omega)\right]\hat{h}_2\hat{h}_2^*\boldsymbol{s}\ .$$

Perpendicularly polarized waves: $\hat{h}_1 \neq 0$, $\hat{h}_3 \neq 0$ and $\hat{e}_2 \neq 0$ (→ Fig. 8.4). The magnetic field strengths follow from the fundamental unknown $\hat{e}_2$ as

$$\hat{h}_1 = \frac{-\gamma_3}{s\mu}\,\hat{e}_2\ , \quad \hat{h}_3 = \frac{\gamma_1}{s\mu}\,\hat{e}_2\ ,$$

while for steady-state uniform planes the energy transfer follows from

$$\boldsymbol{S}_0 = \tfrac{1}{2}\mathrm{Re}\left[\hat{\boldsymbol{e}} \times \hat{\boldsymbol{h}}^*\right] = \tfrac{1}{2}\mathrm{Re}\left[Y(j\omega)\right]\hat{e}_2\hat{e}_2^*\boldsymbol{s}\ .$$

Reflection by and transmission through a plane interface (→ Fig. 4.1):

The incident (plane) wave propagates in medium (1) from the emitter in the positive x_1- and x_3-direction; it is given by

$$\begin{aligned}\hat{\boldsymbol{E}}^i &= \hat{\boldsymbol{e}}^i\exp(-\gamma_1^i x_1 - \gamma_3^i x_3)\ ,\\ \hat{\boldsymbol{H}}^i &= \hat{\boldsymbol{h}}^i\exp(-\gamma_1^i x_1 - \gamma_3^i x_3)\ .\end{aligned}$$

The reflected (plane) wave propagates in medium (1) from the interface in positive x_1-direction and negative x_3-direction; it is given by

$$\begin{aligned}\hat{\boldsymbol{E}}^r &= \hat{\boldsymbol{e}}^r\exp(-\gamma_1^i x_1 + \gamma_3^i x_3)\ ,\\ \hat{\boldsymbol{H}}^r &= \hat{\boldsymbol{h}}^r\exp(-\gamma_1^i x_1 + \gamma_3^i x_3)\ .\end{aligned}$$

The transmitted (plane) wave propagates in medium (2) from the interface in positive x_1- and x_3-direction; it is given by

$$\begin{aligned}\hat{\boldsymbol{E}}^t &= \hat{\boldsymbol{e}}^t\exp(-\gamma_1^i x_1 - \gamma_3^t x_3)\ ,\\ \hat{\boldsymbol{H}}^t &= \hat{\boldsymbol{h}}^t\exp(-\gamma_1^i x_1 - \gamma_3^t x_3)\ ,\end{aligned}$$

For given γ_1^i, in the two media, the components of the propagation vectors perpendicular to the interface follows from (with $\mathrm{Re}[\ ]^{\frac{1}{2}} \geq 0$)

$$\gamma_3^i = \left[(\sigma^{(1)} + s\varepsilon^{(1)})s\mu^{(1)} - (\gamma_1^i)^2\right]^{\frac{1}{2}}\ , \quad \gamma_3^t = \left[(\sigma^{(2)} + s\varepsilon^{(2)})s\mu^{(2)} - (\gamma_1^i)^2\right]^{\frac{1}{2}}\ ,$$

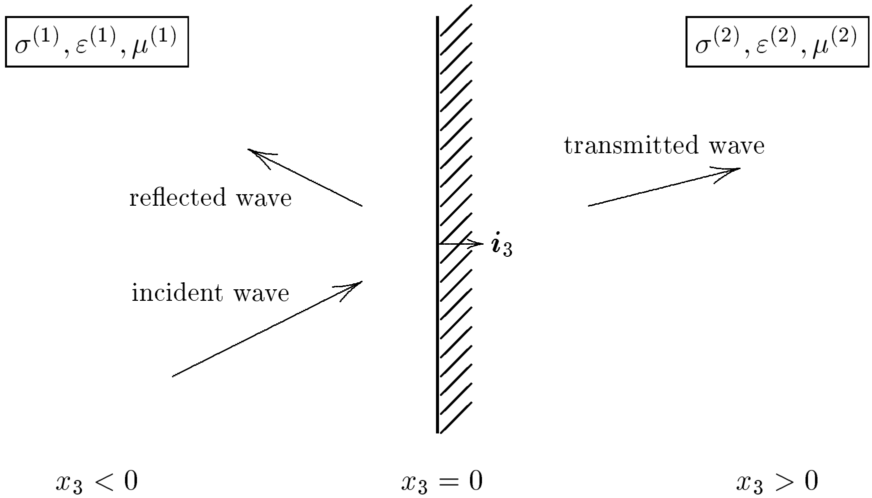


Figure 4.1. Reflection by and transmission through a plane interface.

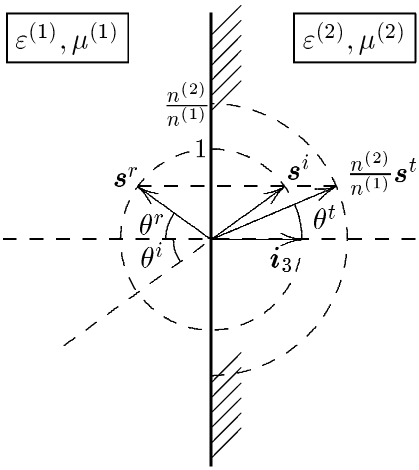
At the interface the electromagnetic field has to satisfy the boundary conditions that the tangential components of the electric and magnetic field strengths are continuous. From this the reflected and transmitted field strengths are expressed in terms of the incident field strengths through the reflection coefficients:

Parallel polarization: $\hat{h}_2^r = R_{||}\,\hat{h}_2^i\,,\ \hat{h}_2^t = T_{||}\,\hat{h}_2^i\,.$

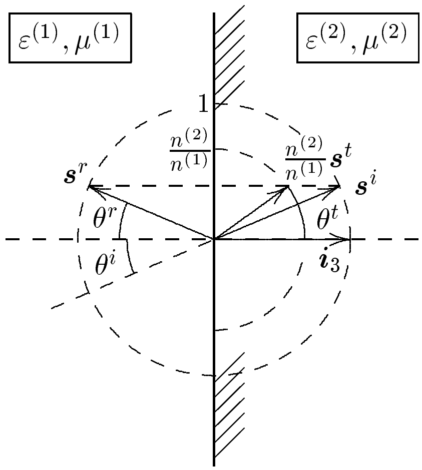
$$R_{||} = \frac{\dfrac{\gamma_3^i}{\sigma^{(1)}+s\varepsilon^{(1)}} - \dfrac{\gamma_3^t}{\sigma^{(2)}+s\varepsilon^{(2)}}}{\dfrac{\gamma_3^i}{\sigma^{(1)}+s\varepsilon^{(1)}} + \dfrac{\gamma_3^t}{\sigma^{(2)}+s\varepsilon^{(2)}}}\,, \quad T_{||} = \frac{2\dfrac{\gamma_3^i}{\sigma^{(1)}+s\varepsilon^{(1)}}}{\dfrac{\gamma_3^i}{\sigma^{(1)}+s\varepsilon^{(1)}} + \dfrac{\gamma_3^t}{\sigma^{(2)}+s\varepsilon^{(2)}}}\,.$$

Perpendicular polarization: $\hat{e}_2^r = R_{\perp}\,\hat{e}_2^i\,,\ \hat{e}_2^t = T_{\perp}\,\hat{e}_2^i\,.$

$$R_{\perp} = \frac{\dfrac{\gamma_3^i}{\mu^{(1)}} - \dfrac{\gamma_3^t}{\mu^{(2)}}}{\dfrac{\gamma_3^i}{\mu^{(1)}} + \dfrac{\gamma_3^t}{\mu^{(2)}}}\,, \quad T_{\perp} = \frac{2\dfrac{\gamma_3^i}{\mu^{(1)}}}{\dfrac{\gamma_3^i}{\mu^{(1)}} + \dfrac{\gamma_3^t}{\mu^{(2)}}}\,.$$



(a) $n^{(1)} < n^{(2)}$



(b) $n^{(1)} > n^{(2)}$

Figure 4.2. Reflection and transmission of a uniform plane wave.

Steady-state uniform plane waves:

Introducing the angles of incidence and reflection (→ Fig. 4.2) we obtain *Snell's law of reflection*,

$$\theta^r = \theta^i \ .$$

Further, in the case of *lossless* media, we introduce the (real-valued) index of refraction $n^{(1)}$ of medium (1) and the index of refraction $n^{(2)}$ of medium (2) as

$$n^{(1)} = c_0[\varepsilon^{(1)}\mu^{(1)}]^{\frac{1}{2}} \ , \quad n^{(2)} = c_0[\varepsilon^{(2)}\mu^{(2)}]^{\frac{1}{2}} \ .$$

With the introduction of the angle of transmission (→ Fig. 4.2) we arrive at *Snell's law of refraction*,

$$n^{(1)}\sin(\theta^i) = n^{(2)}\sin(\theta^t) \ ,$$

provided that the angle of incidence θ^i is less than the critical angle θ^i_c, i.e.,

$$0 \le \sin(\theta^i) \le \sin(\theta^i_c) = \frac{n^{(2)}}{n^{(1)}} \quad \text{for } n^{(1)} > n^{(2)} \ .$$

If $0 \le \theta^i \le \theta^i_c$, the transmitted wave is uniform and the expressions for the *Fresnel reflection and transmission coefficients* become

$$R_{\|} = \frac{\left(\frac{\mu^{(1)}}{\varepsilon^{(1)}}\right)^{\frac{1}{2}} \cos(\theta^i) - \left(\frac{\mu^{(2)}}{\varepsilon^{(2)}}\right)^{\frac{1}{2}} \cos(\theta^t)}{\left(\frac{\mu^{(1)}}{\varepsilon^{(1)}}\right)^{\frac{1}{2}} \cos(\theta^i) + \left(\frac{\mu^{(2)}}{\varepsilon^{(2)}}\right)^{\frac{1}{2}} \cos(\theta^t)} , \quad T_{\|} = \frac{2\left(\frac{\mu^{(1)}}{\varepsilon^{(1)}}\right)^{\frac{1}{2}} \cos(\theta^i)}{\left(\frac{\mu^{(1)}}{\varepsilon^{(1)}}\right)^{\frac{1}{2}} \cos(\theta^i) + \left(\frac{\mu^{(2)}}{\varepsilon^{(2)}}\right)^{\frac{1}{2}} \cos(\theta^t)} ,$$

$$R_{\perp} = \frac{\left(\frac{\varepsilon^{(1)}}{\mu^{(1)}}\right)^{\frac{1}{2}} \cos(\theta^i) - \left(\frac{\varepsilon^{(2)}}{\mu^{(2)}}\right)^{\frac{1}{2}} \cos(\theta^t)}{\left(\frac{\varepsilon^{(1)}}{\mu^{(1)}}\right)^{\frac{1}{2}} \cos(\theta^i) + \left(\frac{\varepsilon^{(2)}}{\mu^{(2)}}\right)^{\frac{1}{2}} \cos(\theta^t)} , \quad T_{\perp} = \frac{2\left(\frac{\varepsilon^{(1)}}{\mu^{(1)}}\right)^{\frac{1}{2}} \cos(\theta^i)}{\left(\frac{\varepsilon^{(1)}}{\mu^{(1)}}\right)^{\frac{1}{2}} \cos(\theta^i) + \left(\frac{\varepsilon^{(2)}}{\mu^{(2)}}\right)^{\frac{1}{2}} \cos(\theta^t)} .$$

It is possible that a Fresnel reflection coefficient vanishes for a particular value of the angle of incidence. The pertaining angle of incidence is called the *Brewster angle.* In the case of parallel polarization, and for dielectric media, where $\mu^{(1)} = \mu^{(2)} = \mu_0$, this Brewster angle follows from

$$\tan(\theta^i_B) = \left(\frac{\varepsilon^{(2)}}{\varepsilon^{(1)}}\right)^{\frac{1}{2}} , \quad \text{(for parallel polarization)} .$$

If $\theta^i > \theta^i_c$, the transmitted wave is non-uniform and we obtain *total reflection*, i.e., $|R_{\|}| = 1$ and $|R_{\perp}| = 1$.

5. Electromagnetic Rays in a Two-dimensional Medium

In a medium that is weakly *inhomogeneous* in the (x_1, x_3)-plane, the concept of electromagnetic rays is useful. We assume that the medium is invariant in the x_2-direction. Then, a twodimensional electromagnetic wavefield exists that is written as

$$\begin{aligned} \hat{\boldsymbol{E}}(x_1, x_3, s) &= \hat{\boldsymbol{e}}(x_1, x_3, s) \exp[-\frac{s}{c_0} L(x_1, x_3)] , \\ \hat{\boldsymbol{H}}(x_1, x_3, s) &= \hat{\boldsymbol{h}}(x_1, x_3, s) \exp[-\frac{s}{c_0} L(x_1, x_3)] , \end{aligned}$$

in which the *eikonal* $L = L(x_1, x_3)$ satisfies the *eikonal equation*:

$$(\partial_1 L)^2 + (\partial_3 L)^2 = c_0^2 \varepsilon \mu .$$