37 Sheet pile walls

An effective way to retain a soil mass is by installing a vertical wall consisting of long thin elements (steel, concrete or wood), that are being driven into the ground. The elements are usually connected by joints, consisting of special forms of the element at the two ends. Compared to a massive wall (of concrete or stone), a sheet pile wall is a flexible structure, in which bending moments will be developed by the lateral load, and that should be designed so that they can withstand the largest bending moments. Several methods of analysis have been developed, of different levels of complexity. The simplest methods, that will be discussed in this chapter, are based on convenient assumptions regarding the stress distribution against the sheet pile wall. These methods have been found very useful in engineering practice, even though they contain some rather drastic approximations.

37.1 Homogeneous dry soil

A standard type of sheet pile wall is shown in Figure 37-1. The basic idea is that the pressure of the soil will lead to a tendency of the flexible wall for displacements towards the left. By this mode of deformation the soil pressures on the right side of the wall will become close to the active state. This soil pressure must be equilibrated by forces acting towards the right. A large horizontal force may be developed at the lower end of the wall, embedded into the soil on the left side, by the displacement. In this part passive earth pressure may develop if the displacements are sufficiently large. The usual schematisation is to assume that on the right side of the wall active stresses will be acting, and below the excavated soil level at the left side of the wall passive stresses will develop. Because the resulting force of the passive stresses is below the resulting force of the active stresses, complete equilibrium is not possible by these stresses alone, as the condition of equilibrium of moments can not be satisfied. Equilibrium can be ensured by adding an anchor at the top of the wall, on the right side. This anchor can provide an additional force to the right. Without such an anchor the sheet pile wall would rotate, until at the extreme lower end of the wall passive earth pressures would be developed on the right side. With an anchor equilibrium can be achieved, without the need for very large deformations. It may be noted that the anchoring force can also be provided by a strut between two parallel walls. This is especially practical in case of a narrow excavation trench.

For the sheet pile wall to be in equilibrium the depth of embedment should be sufficiently large, so that a passive zone of sufficient length can be developed. In case of a very small depth, with a thin passive zone at the toe, the lower end of the wall might be pushed through the soil, with the structure rotating around the anchor point. The determination of the minimum depth of the embedment of the sheet pile wall is an important part of the analysis, which will be considered first. For reasons of simplicity it is assumed that the soil is homogeneous, dry sand. The assumed stress distribution is shown in Figure 37-1.

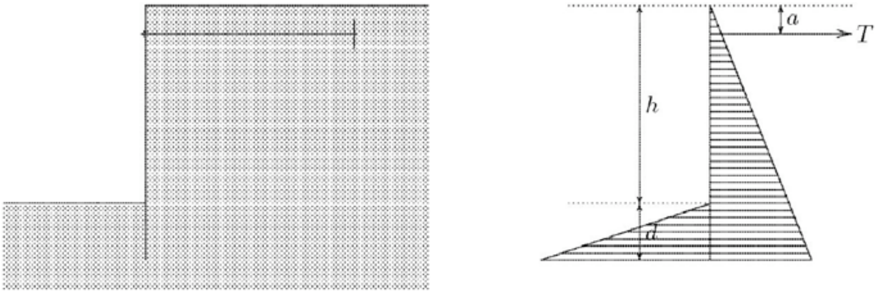

Figure 37-1. Anchored sheet pile wall.

If the retaining height (the difference of the soil levels at the right and left sides of the wall) is h, the length of the toe is d, and the depth of the anchor rod is a, then the condition of equilibrium of moments around the anchor point gives

$$\tfrac{1}{2}K_a\gamma(h+d)^2(\tfrac{2}{3}h+\tfrac{2}{3}d-a)-\tfrac{1}{2}K_p\gamma d^2(h+\tfrac{2}{3}d-a)=0.$$

It follows that

$$(h+d)^2(\tfrac{2}{3}h+\tfrac{2}{3}d-a)=\frac{K_p}{K_a}d^2(h+\tfrac{2}{3}d-a). \tag{37.1}$$

This is an equation of the third degree equation in the variable d. It can be solved iteratively by writing

$$(\frac{d}{h})^2=\frac{2K_a}{3K_p}(1+\frac{d}{h})^2\frac{1+(d/h)-\frac{3}{2}(a/h)}{1+\frac{2}{3}(d/h)-(a/h)}. \tag{37.2}$$

Starting from an initial estimate, for example $d/h=0$, ever better estimates for d/h can be obtained by substituting the estimated value into the right hand of eq. (37.2). This process has been found to iterate fairly rapidly. About 10 iterations may be needed to obtain a relative accuracy of 10^{-6}. The results for a series of values of K_p/K_a and a/h are recorded in Table 37-1.

The magnitude of the anchor force can be determined from the condition of horizontal equilibrium,

$$T=\tfrac{1}{2}\gamma(h+d)^2-\tfrac{1}{2}K_p\gamma d^2. \tag{37.3}$$

The values of T/F_a are given in Table 37-2. The quantity F_a is the total active force,

$$F_a=\tfrac{1}{2}K_a\gamma(h+d)^2. \tag{37.4}$$

a/h	K_p/K_a							
	4	6	8	9	10	12	14	16
0.00	0.793	0.550	0.438	0.401	0.371	0.326	0.294	0.269
0.05	0.785	0.545	0.433	0.396	0.367	0.323	0.290	0.265
0.10	0.777	0.539	0.428	0.392	0.363	0.319	0.287	0.262
0.15	0.768	0.532	0.422	0.386	0.385	0.314	0.282	0.258
0.20	0.759	0.524	0.416	0.380	0.352	0.309	0.278	0.254
0.25	0.749	0.516	0.409	0.374	0.346	0.303	0.273	0.249
0.30	0.737	0.507	0.401	0.366	0.339	0.297	0.267	0.243
0.35	0.724	0.496	0.392	0.358	0.330	0.289	0.260	0.237
0.40	0.710	0.484	0.381	0.348	0.321	0.281	0.252	0.229
0.45	0.693	0.470	0.369	0.336	0.310	0.270	0.242	0.220
0.50	0.674	0.454	0.354	0.322	0.296	0.258	0.230	0.209

Table 37-1: Depth of sheet pile wall, (d/h).

It appears that the anchor carries a substantial part of the total active load, varying from 20 % to more than 50 %. The remaining part is carried by the passive earth pressure, of course.
If the length of the sheet pile wall ($h+d$) and the anchor force are known, the shear force Q and the bending moment M can easily be calculated, in any section of the wall. For the case $K_a = 1/3$, $K_p = 3$ and $a/h = 0.2$ the results are given in Table 37-3. At the location of the anchor the shear force jumps by the magnitude of the anchor force. At the top and at the toe of the wall the shear force and the bending moment are zero.

a/h	K_p/K_a							
	4	6	8	9	10	12	14	16
0.00	0.218	0.244	0.258	0.263	0.267	0.274	0.279	0.283
0.05	0.226	0.254	0.269	0.275	0.279	0.286	0.292	0.296
0.10	0.235	0.265	0.281	0.287	0.292	0.300	0.306	0.310
0.15	0.245	0.277	0.295	0.301	0.306	0.315	0.321	0.326
0.20	0.255	0.290	0.309	0.316	0.322	0.331	0.338	0.344
0.25	0.267	0.305	0.326	0.334	0.340	0.350	0.358	0.364
0.30	0.280	0.321	0.345	0.353	0.360	0.371	0.380	0.387
0.35	0.294	0.340	0.366	0.375	0.383	0.395	0.405	0.413
0.40	0.311	0.361	0.390	0.401	0.409	0.423	0.434	0.443
0.45	0.329	0.386	0.419	0.431	0.441	0.456	0.469	0.478
0.50	0.351	0.415	0.453	0.466	0.478	0.496	0.510	0.521

Table 37-2: Anchor force, (T/F_a).

The largest bending moment, which determines the profile of the pile sheets, is $0.032\gamma h^3$. The results of this example are shown in graphical form in Figure 37-2.
A simple verification of the order of magnitude of the results can be made by considering the sheet pile wall as a beam on two supports, say at $z/h = 0.2$ and at $z/h = 1.2$. The length of the beam then is h, and the average load is $K_a\gamma(0.7h)$. If

this load is thought to be distributed homogeneously along the beam, the maximum bending moment would be $M = qh^2/8 = 0.029\gamma h^3$, which is reasonably close to the true value given above.

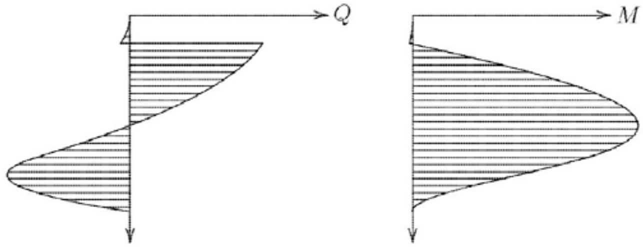

Figure 37-2. Shear force and Bending moment.

If the sheet pile wall is designed on the basis of the maximum bending moment there is no safety against failure. In order to increase the safety of the structure the passive earth pressure is often reduced, by using a conservative value for K_p. The tables then remain valid, but the result will be a somewhat greater length, as can be seen from Table 37-1. If K_p/K_a is taken smaller, the needed value of d/h will be larger. In the next chapter a more advanced method to reduce the risk of failure will be presented.

z/h	$f/\gamma h$	$Q/\gamma h^2$	$M/\gamma h^3$
0.00000	0.00000	0.00000	0.00000
0.10000	-0.03333	-0.00167	-0.00006
0.19999	0.06666	-0.00667	-0.00044
0.20001	0.06667	0.09381	-0.00044
0.30000	0.10000	0.08548	0.00855
0.40000	0.13333	0.07381	0.01654
0.50000	0.16667	0.05881	0.02320
0.60000	0.20000	0.04048	0.02819
0.70000	0.23333	0.01881	0.03119
0.80000	0.26667	-0.00619	0.03184
0.90000	0.30000	-0.03452	0.02984
1.00000	0.33333	-0.06619	0.02483
1.10000	0.06667	-0.08619	0.01699
1.20000	-0.20000	-0.07952	0.00848
1.30000	-0.46667	-0.04619	0.00197
1.38047	-0.68125	0.00000	0.00000

Table 37-3: Sheet pile wall.

An elementary computer program for the calculation of the minimum length of the sheet pile wall, the corresponding anchor force, and the distribution of shear forces and bending moments, is shown as Program 37-1. Input data to this program can be entered interactively. After entering the values of K_a, K_p and a/h the program first calculates the values of the depth of embedment d/h and the anchor force T, and then gives, for an arbitrary value of z/h, to be given by the user, the shear force Q

and the bending moment M. The program can be improved in many ways, especially by adding more advanced forms of input and output, such as graphs of the shear force and the bending moment, to be shown on the screen or on a printer. The implementations of such improvements to the program are left as exercises for the reader.

37.2 Pore pressures

In the previous sections the soil was assumed to be dry, for simplicity. In general the soil may consist of soil and water, however, and the excavation may even contain free water. Thus the general problem of a sheet pile wall should take into account the presence of groundwater in the soil. Because the failure of soils, as described by the Mohr-Coulomb criterion, for instance, refers to effective stresses, the relations formulated above for the earth pressure coefficients K_a and K_p, should be applied to effective stresses only. This means that the vertical effective stresses should be calculated first, before the horizontal effective stresses can be determined. The horizontal total stresses can then be determined in the next step by adding the pore pressure.

```
100 CLS:PRINT "Sheet pile wall in homogeneous dry soil"
110 PRINT "Minimal length":PRINT
120 INPUT "Retaining height ................ ";H
130 INPUT "Depth of anchor ................. ";A
140 INPUT "Active stress coefficient ....... ";KA
150 INPUT "Passive stress coefficinet ...... ";KP
160 PA=KP/KA:A=A/H:B=1/(1.5*PA):D=0:A$="& ###.#####"
170 C=B*(1+D)*(1+D)*(1+D-1.5*A)/(1+D/1.5-A)
180 IF C<0 THEN PRINT "No solution":END
190 C=SQR(C):E=ABS(C-D):D=C:IF E>0.000001 THEN 170
200 PRINT USING A$;"d/h = ";D
210 T=KA*(1+D)*(1+D)/2-KP*D*D/2
220 PRINT USING A$;"T/ghh = ";T
230 INPUT "z/h = ";Z
240 IF Z<0 THEN END
250 IF Z>1+D THEN PRINT " Impossible":GOTO 230
260 F=KA*Z:IF Z>1 THEN F=F-KP*(Z-1)
270 Q=-KA*Z*Z/2:IF Z>A THEN Q=Q+T
280 IF Z>1 THEN Q=Q+KP*(Z-1)*(Z-1)/2
290 M=-KA*Z*Z*Z/6:IF Z>A THEN M=M+T*(Z-A)
300 IF Z>1 THEN M=M+KP*(Z-1)*(Z-1)*(Z-1)/6
310 PRINT USING A$;" f/gh = ";F;
320 PRINT USING A$;" Q/ghh = ";Q;
330 PRINT USING A$;" M/ghhh = ";M
340 GOTO 230
```

Program 37-1: Sheet pile wall in homogeneous dry soil.

The general procedure for the determination of the horizontal stresses is as follows.

1. Determine the total vertical stresses, from the surcharge and the weight of the overlying soil layers.
2. Determine the pore water pressures, on the basis of the location of the phreatic surface. If the pore pressures can be assumed to be hydrostatic (if there is no vertical groundwater flow) these can be determined from the depth below the phreatic surface. Above the phreatic surface the pore pressures may be negative in case of a soil with a capillary rise.
3. Determine the value of the vertical effective stress, as the difference of the vertical total stress and the pore pressure. If the result of this computation is negative, it may be assumed that a crack will develop, as tension between the soil particles usually is impossible. The vertical effective stress then is zero.
4. Determine the horizontal effective stress, using the appropriate value of K_a or K_p at the depth considered, and, if applicable, the local value of the cohesion c.
5. Determine the horizontal total stress by adding the pore pressure to the horizontal effective stress.

The algorithm for this procedure can be summarized as

$$\sigma_{xx} = q_z + \sum \gamma dz, \tag{37.5}$$

$$p = \gamma_w (z - z_w), \quad \text{if} \quad z < z_w - h_c \quad \text{then} \quad p = 0, \tag{37.6}$$

$$\sigma'_{zz} = \sigma_{zz} - p, \quad \text{if} \quad \sigma'_{zz} < 0 \quad \text{then} \quad \sigma'_{zz} = 0, \tag{37.7}$$

$$\sigma'_{zz} = K\sigma'_{zz} \pm 2c\sqrt{K}, \tag{37.8}$$

$$\sigma_{xx} = \sigma'_{xx} + p. \tag{37.9}$$

In these equations it has been assumed that the phreatic level is located at a depth $z = z_w$, and that in a zone of thickness hc above that level capillary water is present in the pores. Above the level $z = z_w - h_c$ there is no water in the pores, which can be expressed by $p = 0$. It has also been assumed, in eq. (37.7), that the particles can not transmit tensile forces. It may also be noted that in computations such as these open water, above the soil, may also have to be considered as soil, having a volumetric weight γ_w. The pore pressures in such a water layer will be found as zero, and the horizontal total stress will automatically be found equal to the vertical total stress. For the analysis of the forces on a wall these forces are essential parts of the analysis. For the analysis of a sheet pile wall the stress calculation must be performed for both sides of the wall separately, because on the two sides the soil levels and the groundwater levels may be different.

An example is shown in Figure 37-3. In this case an excavation of 6 m depth is made into a homogeneous soils. On the right side the groundwater level is located at a depth of 1 m below the soil surface, and on the left side the groundwater level coincides with the bottom of the excavation. For simplicity it is assumed that on both sides of the sheet pile wall the groundwater pressures are hydrostatic. This might be possible if the toe of the wall reaches into a clay layer of low permeability. Otherwise the groundwater pressures should include the effect of a groundwater movement from the right side to the left side. That complication is omitted here. An anchor has been installed at a depth of 0.50 m, at the right side. The length of the wall is initially unknown, but is assumed to be 9 m, for the representation of the horizontal stresses. The soil is homogeneous sand, having a dry volumetric weight of 16 kN/m^3, a saturated volumetric weight of 20 kN/m^3. It is assumed that for this sand $K_a = 0.3333$, $K_p = 3.0$, $c = 0$ and $h_c = 0$.

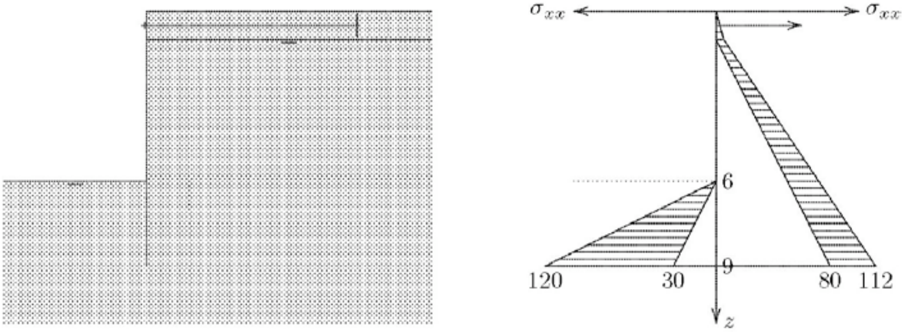

Figure 37-3. Example: The influence of groundwater.

In order to present the stresses against the wand, the simplest procedure is to calculate these stresses in a number of characteristic points. At a depth of 1 m, for instance, at the right side, the vertical total stress is σ_{zz}=16 kPa. Because the pore pressure is zero at that depth the horizontal effective stress is $\sigma'_{xx} = 5.3$ kPa, and the horizontal total stress is equal to that value, because $p = 0$. At a depth of 9 m, the total stress is larger by the weight of 8 m saturated soil, so that $\sigma_{zz} = 176$ kPa. At that depth the pore pressure is $p = 80$ kPa, and the horizontal effective stress is now $\sigma'_{zz} = 96$ kPa. Because $K_a = 0.3333$ the horizontal effective stress is $\sigma'_{xx} = 32$ kPa. Finally, the horizontal total stress is $\sigma_{xx} = 112$ kPa.

At the left side of the wall all stresses are zero down to the level of the bottom of the excavation, at 6 m depth. At a depth of 9 m: $\sigma_{zz} = 60$ kPa and $p = 30$ kPa. This gives $\sigma'_{zz} = 30$ kPa and, because $K_p = 3$, $\sigma'_{xx} = 90$ kPa. The horizontal stress is obtained by adding the pore pressure, i.e. $\sigma_{xx} = 120$ kPa.

Even in this simple case, of a homogeneous soil, the determination of the horizontal loads on the wall is not a trivial problem. In many problems of engineering practice the analysis may be much more complicated, as the soil may consist of layers of different volumetric weight and composition, with variable values of the coefficients

K_a and K_p. This may lead to discontinuities in the distribution of the horizontal stress. The groundwater pressures also need not be hydrostatic. In the case of a permeable soil the determination of the groundwater pressures may be a separate problem.
Another difficulty can be seen in Figure 37-3. Because the water level left of the sheet pile wall is different from the water level on the right, a flow under the sheet pile wall will occur. Because of this the pore pressures around the sheet pile tip will be levelled.
The length of the sheet pile wall is initially unknown. It can be determined by requiring that equilibrium is possible with the toe of the wall being a free end, with $Q=0$ and $M=0$. As in the simple case considered before, see Figure 37-1, the length can be determined from the condition of equilibrium of moments with respect to the anchor point. The simplest procedure is to first assume a certain very short depth of the embedment, with full passive pressures at the left side, then calculating the bending moment at the toe, and then gradually reducing the embedment depth until this bending moment is zero.

The computations can be executed by the Program 37-2. In this program the sheet pile wall is subdivided into a large number of small elements, of length **DZ=H/N**, where **N=NN/3** and **NN=10000**. The horizontal stresses on the right side and the left side are calculated from top to toe, at the same time calculating the moment with respect to the anchor point (this is the variable **MT**). This is done first for the part from to the top to the bottom of the excavation, in lines 220 until 270. The vertical total stresses σ_{zz} to the left and right of the wall are denoted as **TLZ** and **TRZ**, the vertical effective stresses as **SLZ** and **SRZ**, the horizontal effective stresses as **SLX** and **SRX,** and the horizontal total stresses as **TLX** en **TRX**. The quantity **F(I)** is the total distributed load, the sum of the loads from the left and the right. The total length of the wall is gradually increased, from its initial value **HH=H**, in small steps of magnitude **DZ**, until a change of sign of the moment **MT** occurs. Then the length of the wall is known **(HH)**. If at a length of 3 times the excavation depth no equilibrium of moments has been found, the program gives an error statement, and stops. In the course of the analysis the shearing force **Q(I)** and the bending moment **M(I)** are determined, neglecting the anchor force, As soon as the length of the wall is known, the value of the anchor force can be determined, from the condition that at the toe of the wall the bending moment must be zero, see line 360. Then the distributions of the shear force and the bending moment can be corrected for the influence of the anchor force, and the program prints some output data. It also prints the shear force and the bending moment at the toe of the wall. These quantities should be zero. Usually this is not precisely the case, which is an indication of the accuracy.
In the example: **H=6.0**, **DA=0.5**, **CA=0.3333**, **CP=3.0**, **GD=16.0**, **GN=20.0**, **WL=6.0**, **WR=1.0**. The program then gives that the length of the wall should be 11.825 m. The anchor force is 162.710 kN/m, and the maximum bending moment is 544.263 kNm/m. The bending moment at the toe appears to be exactly zero, but the shear force is 0.043 kN. This is a small error, that can be accepted.

```
100 CLS:PRINT "Sheet pile wall in homogeneous soil":PRINT:NN=10000
110 DIM M(NN),Q(NN),F(NN)
120 INPUT "Depth of the excavation (m) ...... ";H
130 INPUT "Depth of the anchor (m) .......... ";DA
140 INPUT "Active stress coefficient ........ ";CA
150 INPUT "Passive stress coefficient ....... ";CP
160 INPUT "Dry weight (kN/m3) ............... ";GD
170 INPUT "Saturated weight (kN/m3) ......... ";GN
180 INPUT "Depth of groundwater left (m) .... ";WL
190 INPUT "Depth of groundwater right (m) ... ";WR
200 N=NN/3:HH=H:DZ=HH/N:DZ2=DZ/2:WW=10:A$="#####.###":PRINT
210 TLZ=0:PL=0:TRZ=0:PR=0:MT=0:Z=0:F(0)=0:Q(0)=0:M(0)=0
220 FOR I=1 TO N:Z=Z+DZ:G=WW:W=WW:IF Z-DZ2<WL THEN G=0:W=0
230 TLZ=TLZ+G*DZ:PL=PL+W*DZ:SLZ=TLZ-PL:SLX=SLZ:TLX=SLX+PL
240 G=GN:W=WW:IF Z-DZ2<WR THEN G=GD:W=0
250 TRZ=TRZ+G*DZ:PR=PR+W*DZ:SRZ=TRZ-PR:SRX=CA*SRZ:TRX=SRX+PR
260 F(I)=TRX-TLX:FF=(F(I)+F(I-1))*DZ2:Q(I)=Q(I-1)-FF
270 M(I)=M(I-1)+(Q(I)+Q(I-1))*DZ2:MT=MT+FF*(Z-DA-DZ2):NEXT I
280 WHILE MT>0:N=N+1:Z=Z+DZ:G=GN:W=WW:IF Z-DZ2<WL THEN G=GD:W=0
290 TLZ=TLZ+G*DZ:PL=PL+W*DZ:SLZ=TLZ-PL:SLX=CP*SLZ:TLX=SLX+PL
300 G=GN:W=WW:IF Z-DZ2<WR THEN G=GD:W=0
310 TRZ=TRZ+G*DZ:PR=PR+W*DZ:SRZ=TRZ-PR:SRX=CA*SRZ:TRX=SRX+PR
320 F(N)=TRX-TLX:FF=(F(N)+F(N-1))*DZ2:Q(N)=Q(N-1)-FF
330 M(N)=M(N-1)+(Q(N)+Q(N-1))*DZ2:MT=MT+FF*(Z-DA-DZ2)
340 IF N=NN THEN PRINT "No solution":STOP:END
350 WEND
360 HH=Z:FT=-M(N)/(HH-DA):Z=0:MM=0
370 FOR I=1 TO N:Z=Z+DZ:IF (Z>DA) THEN Q(I)=Q(I)+FT:M(I)=M(I)+FT*(Z-DA)
380 IF (M(I)>MM) THEN MM=M(I)
390 NEXT I
400 PRINT "Minimum length (m) ............... ";:PRINT USING A$;HH
410 PRINT "Anchor force (kN/m) .............. ";:PRINT USING A$;FT
420 PRINT "Maximum moment (kNm/m) ........... ";:PRINT USING A$;MM
430 PRINT "Shear force at the toe ........... ";:PRINT USING A$;Q(N)
440 PRINT "Moment at the toe ................ ";:PRINT USING A$;M(N)
450 STOP:END
```

Program 37-2: Sheet pile in homogeneous soil, with groundwater.

Again, the computer program has been kept as simple as possible. It can be used as a basis for a more advanced program, with more refined input and output data handling. The input data might be collected in a data file, that can be edited separately, and the output data might be presented in tables or graphs on the screen or on the printer.

Problems

37.1 Verify a number of values in Table 37-1 and Table 37-2, using a computer program.

37.2 Also verify the values from Table 37-3, using a computer program.

37.3 A sheet pile wall is used to retain a height of 5 m, in dry sand, with $\phi = 30^\circ$. The depth of the anchor is 1 m. Determine the minimum embedment depth, according to Table 37-1, and using one of the computer programs.

37.4 Verify the output of the example of Program 37-2. In this case the length of the wall appears to be very large, almost twice the depth of the excavation. What should be the length of the wall if the anchor is located somewhat deeper, say at a depth of 2.0 m?

37.5 Modify the Program 37-2 such that it presents a table of the load, the shear force and the bending moment, as a function of depth.