Exercises on Fracture Mechanics

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Contents

Introduction

Exercises are presented in an order which is roughly determined by the book "Fracture Mechanics" used in the course with the same name. The exercises contain a rather arbitrary mixture ranging from relatively simple theoretical questions to more complex problems for which a large part of the knowledge of the book is required. To facilitate self-study, the answers to most of the exercises are given at the end.

1 Linear-Elastic Fracture Mechanics

1. What is meant by:
   a) the stress concentration factor?
   b) the stress intensity factor?
   c) the critical stress intensity factor?

2. Calculations based on the cohesion force suggest that the tensile strength of glass should be 10 GPa. However, a tensile strength of only 1.5 % of this value is found experimentally. Griffith supposed that this low value was due to the presence of cracks in the glass. Calculate the size $2a$ of a crack normal to the tensile direction in a plate.
   Given: Young's modulus $E = 70$ GPa
   surface tension $\gamma = 0.5$ J/m^2

3. From the energy balance the following fracture criterion can be derived:
$$\frac{\mathrm{d}}{\mathrm{da}}(F - U_{\mathrm{a}}) \geq \frac{\mathrm{d}}{\mathrm{da}}(U_{\gamma}) \quad \text{or} \quad G \geq R\,.$$
   Is this criterion valid for:
   a) Linear-elastic material behaviour?
   b) Non-linear elastic material behaviour?
   c) Elastic-plastic material behaviour?

4. A plate of maraging steel has a tensile strength of 1900 MPa. Calculate the reduction in strength caused by a crack in this plate with a length $2a = 3$ mm oriented normal to the tensile direction.
   Given: Young's modulus $E = 200$ GPa
   surface tension $\gamma_{\mathrm{e}} = 2$ J/m^2
   plastic energy per unit crack surface area $\gamma_{\mathrm{p}} = 2\times10^4$ J/m^2
   critical stress intensity factor $K_{\mathrm{c}} = \sigma_{\mathrm{c}}\sqrt{\pi \mathrm{a}}$

5. Describe the effect of yield strength on fracture behaviour.

6. Consider a plate with an edge crack (see figure). The plate thickness is such that a plane strain condition is present.
   Given: $W = 1000$ mm
   stress intensity factor $K_{\mathrm{I}} = C\sigma\sqrt{\pi a}$ where $C = 1.12$

Material	yield strength σ_{ys} (N/mm^2)	tensile strength σ_{uts} (N/mm^2)	plane strain fracture toughness K_{Ic} (N/mm$^{3/2}$)
Steel 4340	1470	1820	1500
Maraging steel	1730	1850	2900
Al 7075 -T6	500	560	1040

Answer the next questions for the three materials given in the table above:

a) Does fracture occur at a stress $\sigma = ⅔\, \sigma_{ys}$ and a crack length a = 1 mm?
b) What is the critical defect size at a stress $\sigma = ⅔\, \sigma_{ys}$?
c) What is the maximum stress for a crack length a = 1 mm without permanent consequences?

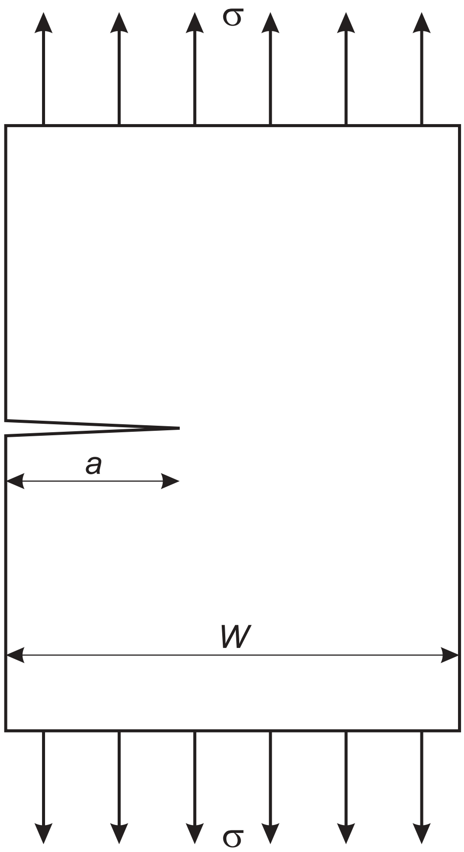


Figure for exercise 6

7. In a mould a wide and 3 mm thick plastic plate is cast. In this plate a central crack is created with a length 2a = 50 mm. The plate is then mechanically tested in the length direction normal to the crack.
   a) If the plate fails at a stress σ = 5 N/mm^2, what then is the K_c value of the material? Corrections for finite specimen size need not be taken into account.
   b) In a similar plate copper wires with a cross section of 2 mm^2 are introduced in the length direction to act as reinforcements. These wires have a relative distance of 20 mm, while one of the wires crosses the central crack exactly through the middle (see figure). By testing the

plate, tensile stresses develop in the parts of the 3 wires located between the crack flanks. These stresses are found to be equal to 24, 36 and 24 N/mm^2 respectively at a test stress $\sigma = 5$ N/mm^2. How high is K_I at this moment if the stresses in the wires are assumed to load the crack flanks over the whole plate thickness?

c) To what level can the test stress σ be increased before failure occurs? Assume that the stresses in the wires are proportional to σ.

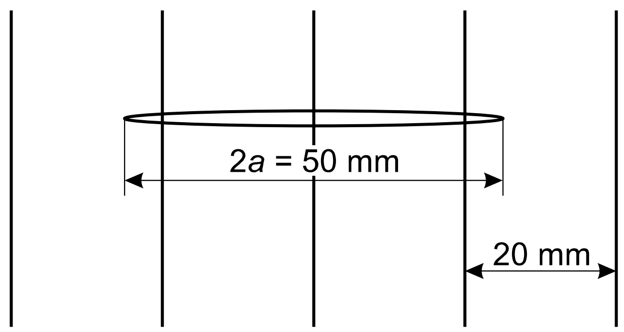


Figure for exercise 7

8. Consider a strip-shaped structural part with a width of 200 mm and a hole in the centre with a diameter $2R = 10$ mm. In service the part is subjected to a tensile load in the length direction of no more than 450.3 kN, while no load is applied directly to the hole.

   Material properties: plane strain fracture toughness $K_{Ic} = 2500$ $N/mm^{3/2}$
   maximum fracture toughness $K_{c\,max} = 4000$ $N/mm^{3/2}$
   yield strength $\sigma_{ys} = 790$ N/mm^2

   a) During the design a safety factor with respect to the yield strength is adopted equal to 2. Furthermore, the presence of the hole is taken into account, but not the stress concentration that is caused by it. What thickness will the structural part be given?
   b) Does this plate thickness lead to K_{Ic}-behaviour according to the ASTM criteria?
   c) Estimate K_c for the plate thickness that is to be applied, using the linear interpolation method proposed by Anderson.
   d) On both sides of the hole cracks may develop due to fatigue. Plot the residual strength as a function of $2(R+a)$, where a is the length of the individual cracks. Assume that $K_I = \sigma\sqrt{\pi(R+a)}$, where σ is the nominal stress.
   e) Determine the critical crack length $2(R+a)$.

9. For the design of a polycarbonate pane the following data are available:
   - at a thickness of 5 mm $K_c = 4$ MPa$\sqrt{m}$;
   - for thicknesses B larger than required for a plane stress state, the value for K_c (in MPa$\sqrt{m}$) is given as a function of B (in m), *i.e.*

$$K_c = K_{Ic} + \frac{3.5K_{c,max}^2}{\pi B \sigma_{ys}^2},$$

where $K_{c,max}$ and K_{Ic} are the critical plane stress and plane strain K_I values respectively.

a) What is the maximum thickness if it is required that fracture is ductile?

b) What happens if a 10× thicker pane is chosen?

c) Compare the ratio of the critical loads (in terms of forces) for the two cases, while assuming a plate geometry containing a central crack with a given size.

Given: plane strain fracture toughness K_{Ic} = 2.2 MPa√m
yield strength σ_{ys} = 64 MPa

10. Consider a 25 mm thick steel specimen with an edge crack. A test is performed in which the specimen is crack line loaded with 10 000 N at the edge of the crack. While artificially increasing the crack size, the crack opening displacement δ at the edge is measured. The following relation between crack length a and displacement δ (both in mm) is found:

$$\delta = 8 \times 10^{-7} a^3 .$$

What is the maximum crack length a, if K_{Ic} for this steel is 1785 N/mm$^{3/2}$?

Material data: Young's modulus E = 210 000 N/mm^2
Poisson's ratio ν = 0.28
yield strength σ_{ys} = 650 N/mm^2

11. Which failure mechanism (besides crack growth) can occur in a thin uni-axially loaded centre cracked plate with a large crack length 2a? Explain this phenomenon.

12. From a small loaded hole cracks have developed (see figure). The hole and the cracks are also subjected to an internal pressure P. Calculate the K-solution, assuming a geometry factor equal to 1.

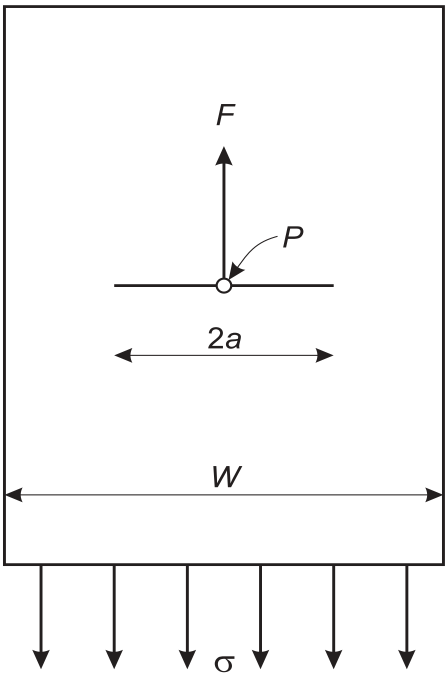


Figure for exercise 12

13. Consider a plate with a width W = 300 mm and a thickness B = 2 mm containing a central crack with a length 2a = 40 mm. The following material data are known:

- Young's modulus E = 210 000 MPa
- yield strength σ_{ys} = 375 MPa

First the plate is loaded with a force of 100 kN.

a) How high will K_I become at least?

At this load the plate does not fail. However the crack appears to have grown to $2a$ = 44 mm.

b) How is this phenomenon called and why has the plate not failed?

c) How high has K_I become?

d) Is the fracture toughness of the material at the current thickness, K_c, higher or lower than K_I? Is the plane strain fracture toughness K_{Ic} higher or lower than K_I?

It is decided to increase the load until failure, which eventually occurs at a force of 120 kN. Using a high-speed movie camera, it is found that just before failure the crack had grown to a length $2a$ = 46 mm.

e) Is this a purely fracture mechanical failure, when using the Feddersen approach? Motivate your answer.

f) Using the available data, which critical K values can be determined for the material under consideration and what are their values?

14. In service a construction part made of a high strength steel is subjected to a constant stress of 1200 MPa. However, in course of time the part fails. Inspection of the fracture surface points to non-stable crack extension from an embedded circular crack, normal to the load direction and with a diameter of 100 μm.

a) How high is the stress intensity factor K_I for this defect as a result of the externally applied load?

It is suspected that the failure is due to hydrogen-induced cracking, *i.e.* a high hydrogen pressure which has developed inside the crack.

b) Using the superposition principle, derive which formula should be used to calculate K_I for this defect due to an internal pressure only.

The fracture toughness for this steel is known: K_{Ic} = 27.5 MPa√m.

c) How high was the hydrogen pressure inside the crack at the moment of failure? For this situation it may be assumed that plane strain conditions are present.

15. Through a hole in a large 10 mm thick plate of polycarbonate a steel nail with a diameter of 3 mm was driven. Afterwards it became obvious that the hole was too small for the nail because cracks developed on opposite sides, each with a length of 1 mm (see figure A). The fracture toughness of the polycarbonate is known: K_{Ic} = 30 Nmm$^{-3/2}$.

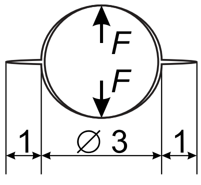


Figure A for exercise 15

a) How high is K_I for these cracks?

b) How high is the load that is exerted by the nail onto the plate? Consider this load to consist of two point forces F (see figure A). Furthermore, assume that these forces cause the same stress intensity as if they were acting on the flanks of a central crack with the same total length, *i.e.* 5 mm.

In spite of the cracks the plate is subjected to a load σ of 5 MPa in the direction normal to the cracks. After loading it is found that both cracks have grown to a size of 3½ mm (see figure B).

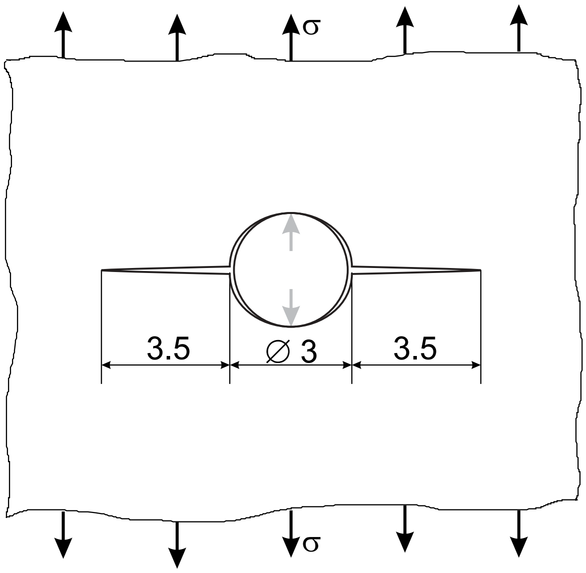


Figure B for exercise 15

c) How high is K_I now?

d) Is the nail still attached to the plate?

16. The total energy U of a loaded elastic body containing a crack is given by:

$$U = U_o + U_a + U_\gamma - F\ .$$

a) What do the terms mean?

b) What is the energy available for crack growth?

c) What is the energy required for crack growth?

17. According to Irwin plastic deformation at the crack tip results in an apparently longer crack, *i.e.* $K_I = \sigma\sqrt{\pi(a + r_y)}$, where r_y is the radius of the plastic zone.
What will the K_I solution become if we realise that K_I affects the size of r_y, while r_y also affects the size of K_I?

18. Plot how the critical stress intensity K_c depends on the thickness and explain this.

19. For a double cantilever beam specimen loaded by a force P, the displacement v can be written as:

$$v = \frac{2Pa^3}{3EI},$$

where a = crack length,
E = Young's modulus,
I = moment of inertia.

A constant displacement is applied to the specimen. Derive whether K_I increases or decreases when the crack grows.

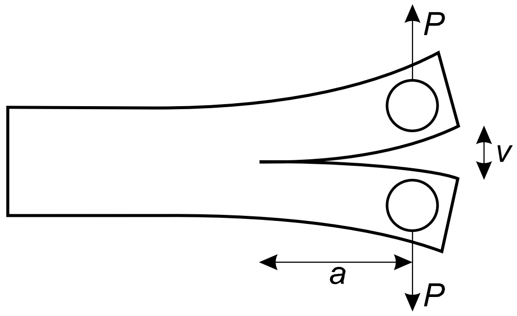


Figure for exercise 19

20. Consider a plate with a width W = 300 mm and a thickness B = 12.5 mm containing a central crack with a size $2a$ = 50 mm. In a test the load at failure is found to be 450 kN.
   a) How high is the fracture toughness K_e in MPa√m? Is this a valid value?
   b) What is the minimum plate width for a valid K_e determination?
   c) Calculate the residual strength (in MPa) for a plate with the same crack length and thickness but with a width W = 100 mm.
   d) If this was the result of a test, what would K_e be? Is this a valid value?

   Given: geometry factor $f(a/W) = 1$
   yield strength σ_{ys} = 500 MPa

21. In a K_c (K_e) test the plate fails at a stress σ_c higher than $\frac{2}{3} \cdot \sigma_{ys}$. If this stress is used to calculate K_c or K_e, will the result be too low or too high? Explain the answer.

22. Show that for crack growth under constant load conditions half of the work performed on a centre cracked plate is used for an increase of the elastic energy of the plate, while the other half is used for crack extension.

23. The through-thickness yielding criterion is: $K_{Ic} \geq \sigma_{ys}\sqrt{B}$
   a) What is the use of this criterion?
   b) Explain why at 30 mm thickness 10Ni steel fails in a tougher way than the alloy Ti-6Al-V.

24. The R-curve for a certain material can be expressed as:

$$R = \frac{K_{\mathrm{Ic}}^{\;2}}{E} + 3(a - a_{\mathrm{o}})^{0.1},$$

with R in MN/m, a and a_{o} in meters, $K_{\mathrm{Ic}} = 150$ MPa√m and $E = 210\,000$ MPa.
A wide centre cracked plate of this material has a crack length $2a_{\mathrm{o}} = 60$ mm.
a) Show that this plate allows a maximum stable crack growth of 3.1 mm at both tips.
b) Calculate the critical stress (σ_{c}) and the fracture toughness (K_{c})?

25. On the fracture surface of a plate-shaped part of aluminium alloy 7075-T6 that has failed due to an overload, shear lips are found. The shear lips on each side have a width of 1 mm and so in total 2 mm of the fracture surface in the thickness direction is slant. The thickness of the part is 4 mm.
Calculate K_{c} for this material at this thickness using the results of Knott's analysis of the model of Krafft *et al.* (Young's modulus $E = 70\,000$ N/mm^2)?

26. There is a choice between two materials, designated as A en B. A construction part is being designed with a safety factor 3 with respect to the yield strength. Which material should be chosen if the maximum allowable crack size is to be as large as possible?

Given: material A: $\sigma_{\mathrm{ys}} = 350$ MPa ; $K_{\mathrm{Ic}} = 50$ MPa√m
material B: $\sigma_{\mathrm{ys}} = 500$ MPa ; $K_{\mathrm{Ic}} = 60$ MPa√m

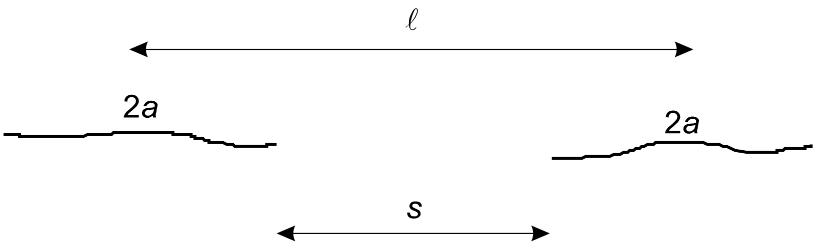


Figure for exercise 27

27. For a through-thickness crack (length $2a$) in a wide plate $K_{\mathrm{I}} = \sigma\sqrt{\pi a}$, whereas for a row of collinear cracks with interspacing ℓ (see figure)

$$K_{\mathrm{I}} = \sigma\sqrt{\ell \tan\frac{\pi a}{\ell}}\,.$$

In guidelines for the safety of welded constructions, requirements are set for plane crack-like welding defects with respect to the distance s (see figure). If s is smaller than the defect size $2a$, two neighbouring defects should be considered as one larger defect (length $s + 4a$). In this case the first K-formula is used ($K_{\mathrm{I}} = \sigma\sqrt{\pi a}$).
a) Determine the ratio between the two K_{I} values for $s = 2a$.
b) Is this guideline conservative, *i.e.* on the safe side, for $s = 2a$?

c) Calculate the ratio between the two K_I values for a very small s value, *i.e.* $s = a/10$, and explain the result.

28. A tool (pick-axe) is used to break up old roads (see figure).
   a) What force can be allowed without a crack? Use the fact that stresses acting in the same direction on the same plane may be superposed. Take the yield strength as a failure criterion. Perform the calculation for the part where a crack is drawn in the figure.
   b) A crack develops, but it is only detected when it reaches a size of 5 mm. What is the critical fracture force at that moment?

Given: fracture toughness $K_{Ic} = 59$ MPa√m
yield strength $\sigma_{ys} = 1500$ MPa
correction for a free surface: tensile 1.12, bending 1.03
correction for finite dimensions is not necessary
bending stress $\sigma_b = {}^{6M}/_{Bh^2}$, where M is the bending moment

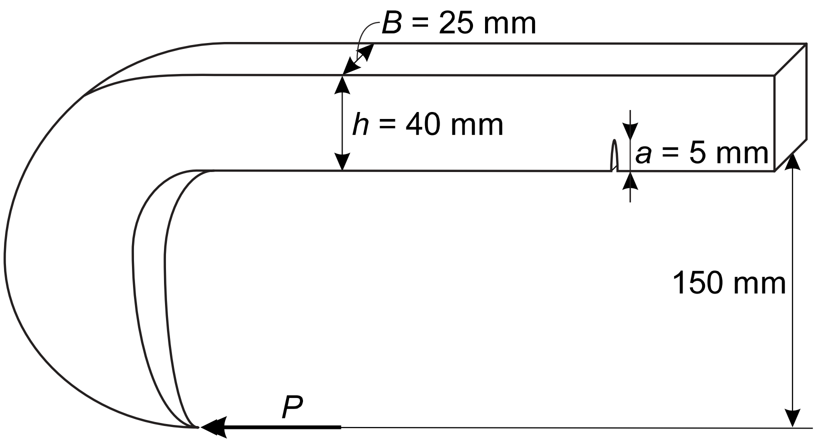


Figure for exercise 28

29. In a plate of aluminium alloy 7075 a central defect is present with a total length of 3 mm.
   a) Calculate the critical stress for fracture using the Griffith criterion and using K_{Ic}.
   b) Explain possible differences.

Given: surface tension $\gamma_e = 1.14\times10^{-6}$ J/mm^2
Young's modulus $E = 7\times10^4$ N/mm^2
fracture toughness $K_{Ic} = 1040$ N/mm$^{3/2}$

2 Elastic-Plastic Fracture Mechanics

30. A J_{Ic} test is performed on steel with the following properties:

$$E = 207 \text{ GPa}; \ \sigma_{ys} = 360 \text{ MPa en } \sigma_{uts} = 560 \text{ MPa} \ ; \ \nu = 0.28.$$

For this purpose a 3-point bend specimen is used with the following dimensions:

$$W = 50 \text{ mm}; B = 20 \text{ mm}; a = 30 \text{ mm}.$$

The load is found to increase linearly with displacement. At the onset of crack extension the load is 25 kN, while the displacement is 4 mm.

a) What value follows for J_{Ic}, if this is defined as J at the onset of crack extension?
b) Is this value acceptable according to ASTM standard E 813?
c) What is the corresponding value for K_{Ic}?
d) What thickness should the specimen be for a valid K_{Ic} determination?

31. For a large welded steel vessel the following data are available:

service stress $\sigma = 200$ N/mm^2
yield strength weld metal $\sigma_{ys} = 450$ N/mm^2
Young's modulus weld metal $E = 205\,000$ N/mm^2

Due to shrinkage after the welding process, residual stresses develop in the weld seam which can be as high as the yield strength of the weld metal. By means of a heat treatment these stresses can be reduced. It is required that the vessel can withstand a crack in the weld metal with half crack size $a = 60$ mm. Furthermore the CTOD value in this case may not exceed 0.5 mm.

To what level should the residual stresses be reduced to comply with these requirements?

32. A J_{Ic} test is performed on HY130 steel. The results, measured on SENB specimens, are:

U [J]	Δa [mm]
45	0.10
70	0.40
85	0.67
105	0.99
117.5	1.22

Specimen dimensions: span $L = 4W = 200$ mm
width $W = 50$ mm
thickness $B = 0.5\ W = 25$ mm
crack length $a = 30$ mm

Material properties: yield strength $\sigma_{ys} = 925$ MPa
tensile strength $\sigma_{uts} = 953$ MPa
Young's modulus $E = 210\,000$ MPa
Poisson's ratio $\nu = 0.28$
mass density $= 7.8\times10^3$ kg/m^3

a) Approximately (!) determine J_{Ic} using a spreadsheet or on paper. Assume that conditions which can not be checked but are necessary for a valid determination are fulfilled.
b) How high is K_{Ic} for this steel?
c) At least how many kilograms of weight would be saved by determining K_{Ic} through J_{Ic} with this test? Assume that the ratios of the dimensions of the K_{Ic} specimen are equal to those listed above.

33. On a bend specimen a single-specimen J_c test is performed. Using a non-destructive technique crack initiation is observed at a load P = 9315 N. During the test the load is recorded as a function of the load-point displacement v. A polynominal fit of the P-v curve resulted in:

$$v = 10^{-5}P + 10^{-14}P^4 \ ,$$

where v is expressed in mm and P in N.
a) Determine the critical J value at initiation.
b) Is the plate thickness sufficient for a valid J_{Ic} test?

Given: width $W = 50$ mm
crack length $a = 12$ mm
thickness $B = 28$ mm
flow stress $\sigma_o = 1000$ MPa

34. Derive: $J = 2\sigma_o\Delta a$
How is this J versus Δa line called and what is its purpose?

35. For materials with a moderate toughness (*e.g.* aluminium alloys) K_{Ic} can be determined from J_{Ic}. Express the minimum required thickness for the J_{Ic} test (B_j) in terms of the minimum required thickness for the K_{Ic} test (B_k).

Given: Young's modulus $E = 70\,000$ MPa
yield strength $\sigma_{ys} = 345$ MPa
tensile strength $\sigma_{uts} = 500$ MPa
E' for plane strain = E (for plane stress)

36. For the derivation of the J-integral the so-called deformation theory of plasticity is used.
What does this theory mean and which important condition (limitation) results from this?

37. Show that the occurrence of plastic constraint in actual structural parts leads to more safety when using the COD design curve.

38. A (non-linear) elastic plate is loaded, leading to the load-displacement diagram shown in the figure.
    a) Assume that no crack growth occurs during the loading process and indicate graphically the part of the area representing the change in potential energy of:
       - the specimen,
       - the combination of the specimen and the loading system.
    b) Now assume that crack growth occurs under constant load conditions. Answer the previous question but now for the change in potential energy due to crack growth.

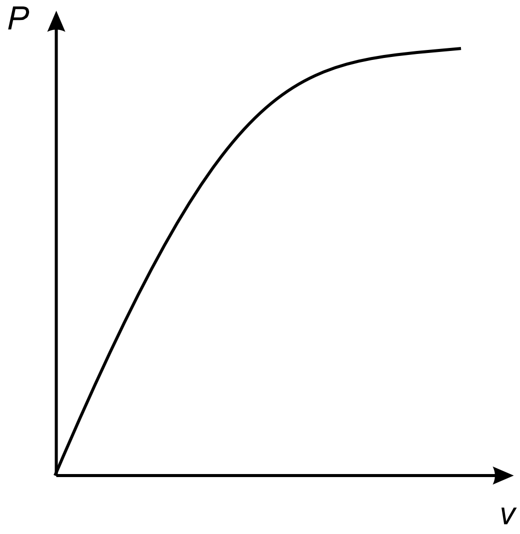


Figure for exercise 38

39. Show that in the linear elastic regime the COD design curve predicts a maximum permissible crack length (a_{max}) equal to half the critical crack length calculated with LEFM.

40. In relation with the J integral the expressions $\mathrm{d}x_1 = -n_2\mathrm{d}s$ and $\mathrm{d}x_2 = n_1\mathrm{d}s$ are used. What is the meaning of these quantities? Derive these formulas.

41. Explain why in case of short cracks or of very long cracks the linear elastic fracture criteria are no longer valid, even for relatively brittle materials.

42. The value of the J integral is independent of the exact path followed surrounding the crack tip in counterclockwise direction, starting on the lower and ending on the upper crack flank.
    a) What is J for a closed contour, *i.e.* one not surrounding the crack tip singularity?
    b) Indicate what is wrong in the following reasoning:
       Along the closed contour ABPA shown in the figure, the J integral is zero. Along the flanks AP and BP J is zero too. Consequently J must be zero along the contour surrounding the crack tip, A → B.

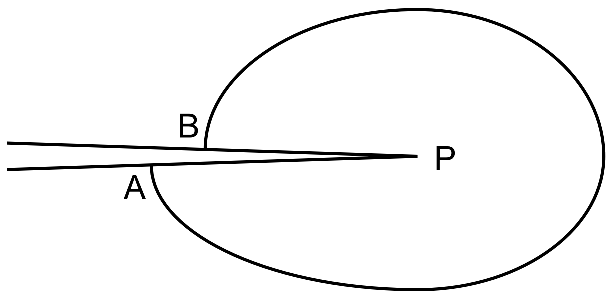


Figure for exercise 42

3 Fatigue and Stress Corrosion Crack Growth and Dynamic Effects

43. Describe the possible effects of a peak load on the crack growth behaviour during variable amplitude fatigue.

44. A crack with a depth of 2 cm is found in a cast iron rod of an old steam engine. Every weekend the machine is used for 8 hours for demonstration purposes, during which it is run at 15 rpm and the load in the rod varies from $+6.4\times10^4$ N to -6.4×10^4 N. It may be assumed that the crack is completely closed under compressive load but that under tensile load crack closure is negligible. The Paris relation is sufficiently accurate until failure.
    a) Is it safe to use this machine for demonstrations and for how long?
    b) Answer these questions assuming a load variation from +0.92 MN to −0.92 MN.

    Given: $K_c = 16$ MPa√m
    $\Delta K_{th} = 5$ MPa√m
    cross section rod = 0.04 m^2
    factor in the Paris relation: $C = 4.3\times10^{-8}$ $m(MPa\sqrt{m})^{-4}$
    $K_I = 1.12\,\sigma\sqrt{\pi a}$

45. Which of the two cases (A or B) drawn in the figure gives more retardation of the fatigue crack growth and why is this?

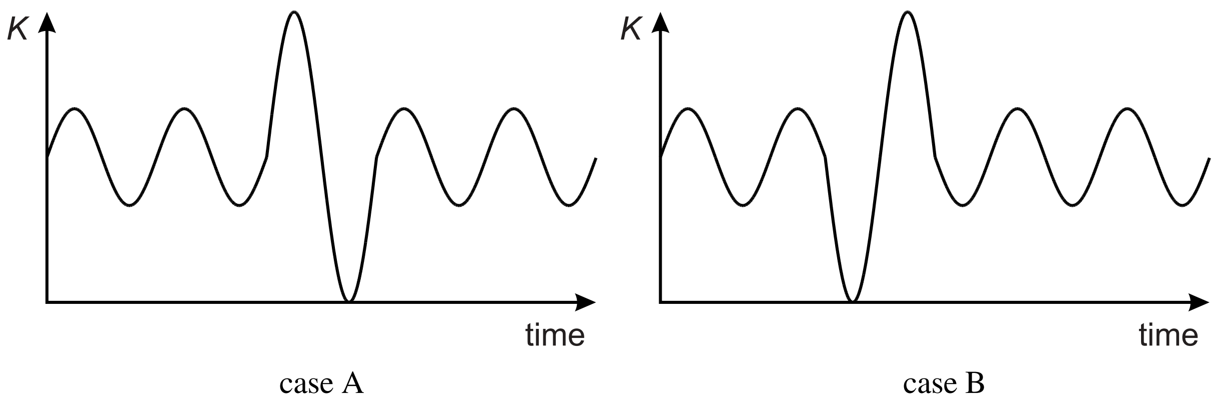


Figure for exercise 45

46. On a material a constant amplitude fatigue load with $\sigma_{max} = 100$ MN/m^2 and $R = 0$ is applied. An edge crack is growing in the material. At a crack length of 16 mm there is a single overload with $\sigma_{max} = 200$ MN/m^2.
    Until what crack length does Wheeler's model predict an effect on da/dn due to the overload?

    Given: yield strength $\sigma_{ys} = 400$ MN/m^2

stress intensity factor $K_I = \sigma\sqrt{\pi a}$
plastic constraint factor $C = 1$

47. Divers detect a through-thickness crack with length $2a$ = 70 mm in a support of an oil-winning platform in the North Sea. The crack is situated along a weld below the water-line. Relevant dimensions are given in the figure.
The constant amplitude loading exerted by the water waves is $\Delta\sigma$ = 15 N/mm^2, with a stress ratio R = 0 and a frequency f = 1/6 Hz. For this geometry it may be assumed that $K = \sigma\sqrt{\pi a}$. The steel of the support (BS 4360-50D) permits a maximum crack size of $2a$ = 150 mm at the maximum load of the waves. Further material data are:
fatigue threshold ΔK_{th} = 60 N/mm$^{3/2}$
Paris relation: $da/dn = 5.1\times10^{-11}\ \Delta K^{\,2.53}$ mm/cycle with ΔK in N/mm$^{3/2}$

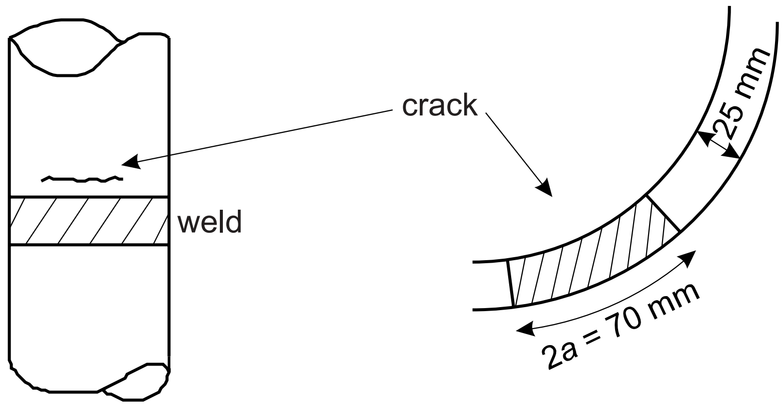


Figure for exercise 47

Only in the summer period, from April 1 to October 1, it is allowed to dive in the North Sea. It is June 1st when the crack is detected. Calculate if it is necessary to repair the crack before October or if it is possible to wait until next year.

48. During stress corrosion tests with decreasing K specimens corrosion products can lead to apparently lower K_{Ith} values.
a) Explain this phenomenon.
b) How can we measure if it has had an effect on the measured K_{Ith}?

49. A wide plate with a central crack is subjected to a constant amplitude fatigue load with $\Delta\sigma$ = 100 MPa and R = 0. At a crack length $2a$ = 6 mm an overload occurs with a stress of 170 MPa. How high is the crack growth rate immediately after the overload, after 0.2 mm and after 0.4 mm of crack growth (at each tip)? Assume plane stress conditions and, for convenience, consider K to remain constant during the indicated crack growth intervals.

Given: Wheeler exponent m = 1.5
yield strength σ_{ys} = 420 MPa

Paris relation: $\mathrm{d}a/\mathrm{d}n = 1.5\times10^{-10}(\Delta K)^4$ mm/cycle with ΔK in MPa√m

50. Explain why a rapidly propagating crack in a pressurised pipeline will sooner show arrest if the pipeline is fluid-filled than if it is gas-filled?

51. Consider the following plot of G,R versus crack length.

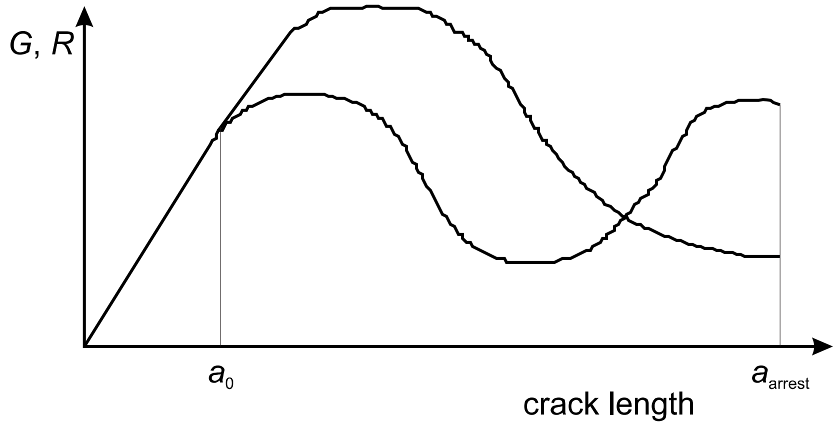


Figure for exercise 51

Indicate in the graph where the crack will grow accelerated and where it will slow down.

52. The following striation patterns are found on a fatigue fracture surface using a scanning electron microscope (SEM).

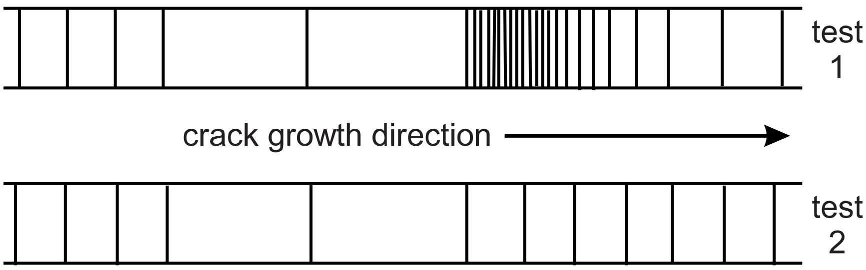


Figure for exercise 52

Sketch a possible K variation with crack length for both tests. The frequency may be assumed the same for both cases and independent of crack length.

53. In the context of fatigue crack growth, what is meant by delayed retardation?
Describe the conditions for which this occurs.

54. Show that the reversed plastic zone in fatigue is about ¼ of the normal (monotonic) plastic zone size. Hint: take $R = 0$.

55. During stress corrosion crack growth tests using decreasing K specimens, crack tip blunting and/or crack tip branching leads to apparently higher K_{Ith} values. Explain both phenomena.

56. During a fatigue crack growth test on aluminium a transition takes place in the applied ΔK, as is indicated in the figure.
   a) How high is R (approximately) before and after the transition?
   b) Draw a line in the figure, representing the value of K_{op} (both before and after the transition). Use a linear crack closure relation for this purpose.
   c) Indicate qualitatively how the crack growth rate ${}^{da}/_{dn}$ depends on time.

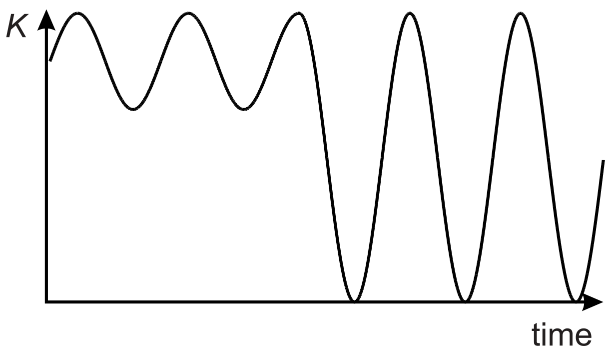


Figure for exercise 56

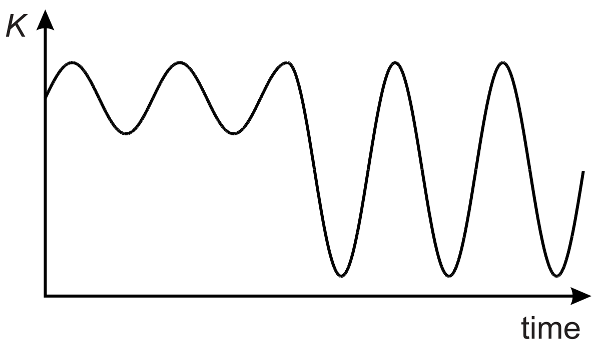
57. A structural part of about 20 cm thickness is subjected to a tensile load that varies 5 times per minute between 110 and 200 MPa. For the material the following data are known:
   - $K_{Ic} = 30\ \text{MPa}\sqrt{\text{m}}$
   - $\Delta K_{th} = 5\ \text{MPa}\sqrt{\text{m}}$
   - Paris curve: ${}^{da}/_{dn} = 2\times10^{-11}(\Delta K)^3$ with ${}^{da}/_{dn}$ in ${}^{\text{m}}/_{\text{cycle}}$ and ΔK in $\text{MPa}\sqrt{\text{m}}$

   a) Based on these data, what can be concluded about the lifetime of the part?

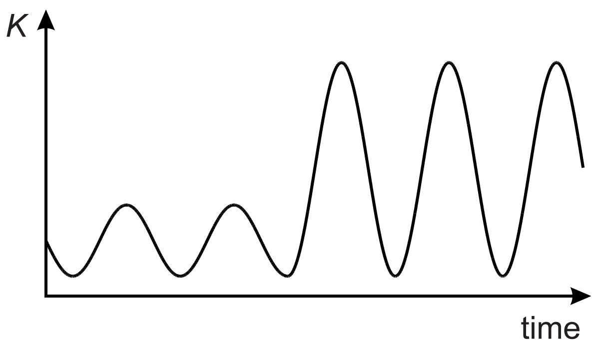
   Investigations show that after the manufacture of this type of parts semi-circular surface cracks may be present. Using an eddy current technique such cracks can be found if they are at least 2 mm deep.

   b) Calculate whether or not there is a risk for crack growth.
   c) What is the maximum inspection interval that ensures no failure will occur?

58. During fatigue loading of an aluminium specimen transitions take place as drawn in the figures, *i.e.* the frequency remains the same, but K_{min} (case A) respectively. K_{max} (case B) change.
   Draw qualitatively how ${}^{da}/_{dn}$ depends on crack length directly after the load transition for both cases. Take the effect of crack closure into account.



case A



case B

Figure for exercise 58

59. A cylindrical steel pressure vessel, with a diameter of 7.5 m and a wall thickness of 40 mm, must operate at a pressure of 5.1 MNm^{-2}. In the design it is assumed, that the vessel fails as a result of unstable extension of a fatigue crack developing in the axial direction of the vessel. To avoid such a failure, the number of times the vessel may be loaded from zero to maximum load is limited to 3000.

The maximum allowable K (at failure) in this material is 200 $MNm^{-3/2}$ and fatigue crack growth is described by:

$$\mathrm{d}a/\mathrm{d}n = A(\Delta K)^4 \text{ , where } A = 2.44\times 10^{-14}\ (\mathrm{MNm}^{-3/2})^{-4}\mathrm{m}.$$

Determine the pressure at which the vessel should be tested before use in order to guarantee that failure does not occur before loading it 3000 times. Assume $K_\mathrm{I} = \sigma\sqrt{\pi a}$.

4 Combinations & Others

60. In fracture mechanics inclusions are generally subdivided in three categories classified according to their size. The largest category is that of inclusions ranging from 0.5 to 50 μm. What role do these inclusions play in the following phenomena?
    a) Ductile fracture
    b) Initiation of a fatigue crack in:
        1. smooth specimens
        2. notched specimens
    c) Fatigue crack growth at:
        1. low crack growth rates
        2. high crack growth rates
    d) Fracture toughness K_{Ic}.

61. A bar containing a crack with a length a_o = 0.002 m is subjected to a fatigue load. For this geometry $K_I = 0.8\sigma\sqrt{\pi a}$ for all crack sizes.
    a) Calculate the crack length after the bar has been subjected to 6000 cycles between a stress of 60 and 120 N/mm^2 and next to 3000 cycles between 0 and 120 N/mm^2.
    b) Calculate the crack length if the loading order is reversed.
    c) Assume the bar with the original crack was subjected to a static load of 160 MPa in an environment that leads to a $K_{I_{scc}}$ of 6 MN/m$^{1.5}$. Would stress corrosion cracking occur?

    Given: Paris relation: $da/dn = 10^{-10}(\Delta K)^4$ with a in meters and ΔK in MN/m$^{1.5}$
    fatigue threshold ΔK_{th} = 5 MN/m$^{1.5}$

62. A long cylindrical pressure vessel is being designed. In practise the vessel will experience internal (gas) pressure variations between 0 and 3.5 MPa. It is recognised that under these circumstances a half circular fatigue crack can extend from the interior to the exterior, after which leakage will occur.
    What is the minimum wall thickness the vessel should have to ensure the crack stops growing due to the pressure drop caused by leakage before (unstable) failure of the vessel can occur?

    Given: external cylinder diameter = 1 m
    fracture toughness K_c = 49.4 MPa√m
    Hints: Neglect the wall thickness relative to the diameter of the vessel, assume that the crack remains half circular during extension until leakage and take $K_I = \sigma\sqrt{\pi a}$ (solution for a through-thickness crack).

63. Cracks develop on opposite sides of an unloaded central hole in a tensile element. The plate

width is 195 mm, the plate thickness is 6 mm and the hole diameter is 15 mm. In service the load varies 100 times per day between 0 and 351 000 N. The minimum detectable crack length, a, is 1 mm. For this geometry it is safe to assume that $K_I = \sigma\sqrt{\pi(a+r)}$, where r is the radius of the hole.

The material has a K_{Ic} of 2000 $N/mm^{3/2}$ and reaches a maximum K_c of 3400 $N/mm^{3/2}$ at small thicknesses. Furthermore, for a loading ratio $R = 0$ the crack growth rate $da/dn = 10^{-13}(\Delta K)^4$ mm/kilocycle, with ΔK in $N/mm^{3/2}$. The yield strength, σ_{ys}, is 800 N/mm^2.

a) Estimate K_c for the thickness to be applied using Anderson's model.
b) In what range of a-values will the plate show K_c behaviour according to Feddersen?
c) Determine the inspection interval if at least three inspections should fall within the crack growth period.

64. Investigation of a failed structural part leads to fatigue as the cause. The fatigue crack depth is 5 mm, after which failure occurred. On the fracture surface striations are found: the striation distance at the end of the fatigue crack is 2 mm at a magnification of 2000×.

The fatigue load the structural part has been subjected to, is a constant load amplitude with a load ratio $R = 0$, but with an occasional overload cycle with a maximum load of twice the maximum of the constant amplitude load.

Material tests show that the yield strength $\sigma_{ys} = 900$ MPa and that fatigue crack growth at constant amplitude and $R = 0$ is described by $da/dn = 6\times10^{-12}\times\Delta K^3$, with da/dn in m/cycle and ΔK in MPa√m. This relation may be assumed valid until fracture and crack closure can be ignored. For this structural part $K_I = \sigma\sqrt{\pi a}$.

a) Calculate the fracture toughness K_c (in MPa√m) of the material.
b) Calculate the maximum stress (in MPa) of the constant amplitude load.

65. Consider a DCB specimen (see figure). With the bolt a displacement is applied of 0.1 mm. After that the specimen is placed in an environment that causes stress corrosion in the material.

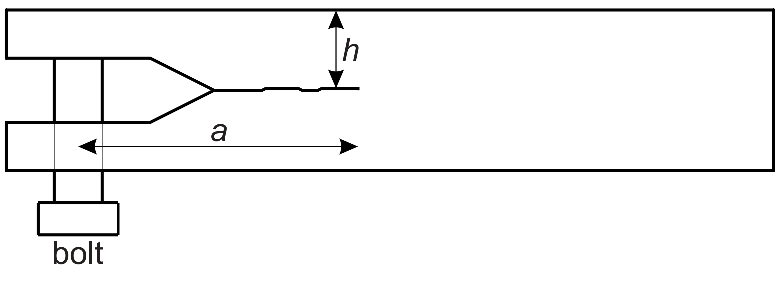


Figure for exercise 65

How high is K_{Iscc} if the crack stops growing at $a = 50$ mm and plane stress conditions prevail?

Given: Young's modulus $E = 70\,000$ N/mm^2
height $h = 25$ mm

66. A 300 mm wide and 11 mm thick plate is subjected to a fatigue load at a frequency of 1 Hz. The stress varies between σ_{max} = 120 MPa and σ_{min} = 40 MPa. At a certain moment an edge crack is found of 4.5 mm.

    Calculate how many hours it takes before the plate fails.

    Material data: Paris relation: $da/dn = 3.5\times10^{-10}\Delta K^{2.5}$ m/cycle with ΔK in MPa√m
    yield strength σ_{ys} = 381 MPa
    plane strain fracture toughness K_{Ic} = 35 MPa√m
    fracture toughness for thin plate (≈ 1 mm) K_c = 62 MPa√m

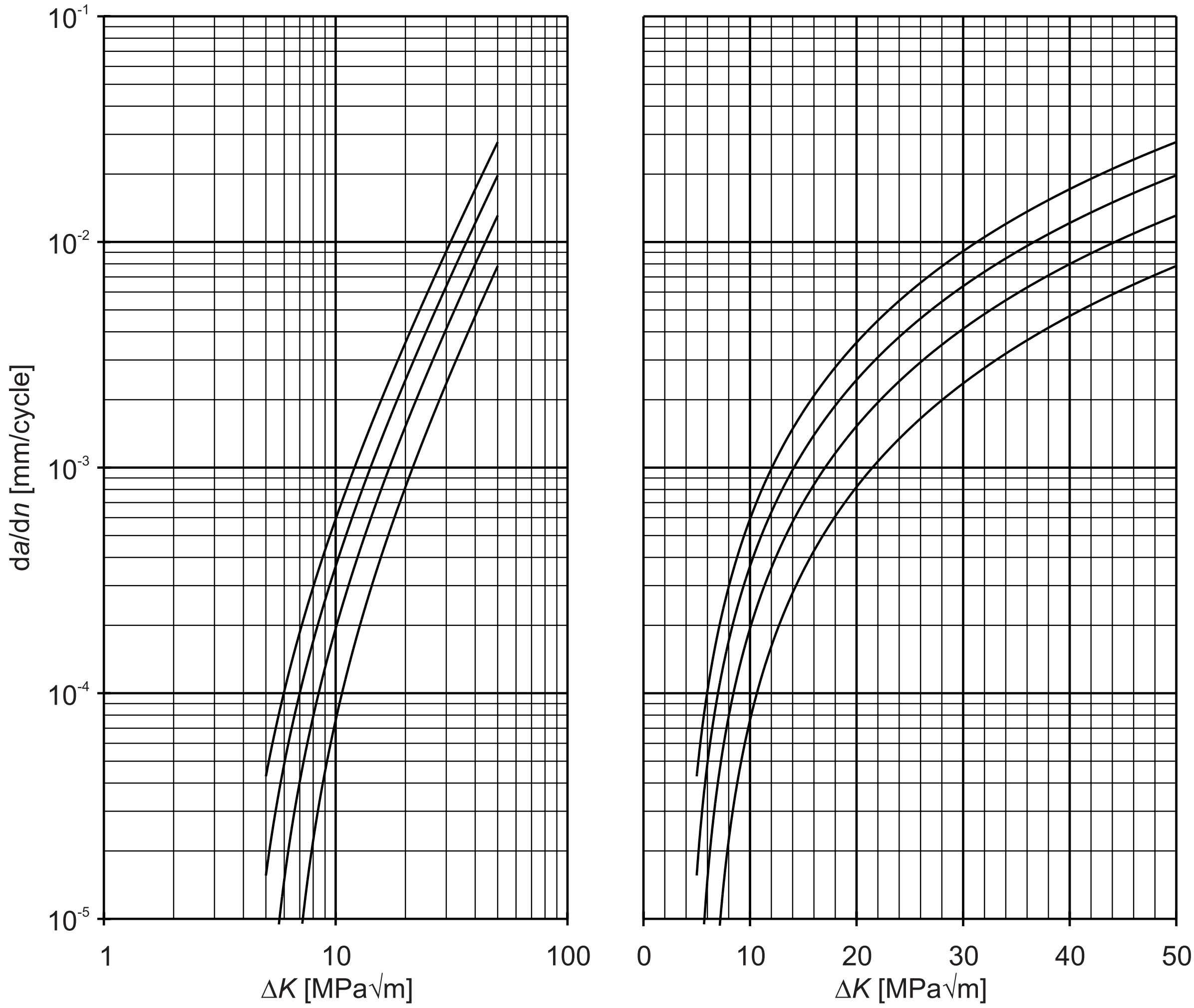
67. Failure analysis showed that a construction part, which was subjected to a constant load, had failed by stress corrosion cracking. The crack length at failure was 6 mm. The material thickness was sufficient for plane strain. The factory where the parts are made checks every sample using non-destructive techniques. It is guaranteed that no crack-like defects larger than 1 mm are present on delivery.

    Calculate the percentage with which the thickness of the part should be increased to avoid future failures due to stress corrosion cracking.

    Given: K_{Ic} = 60 MPa√m
    $K_{I_{scc}}$ = 20 MPa√m
    $K_I = 1.12\,\sigma\sqrt{\pi a}$

68. A wide plate contains a central crack. The plate is subjected to constant amplitude fatigue loading until failure. Analyses of the fracture surface reveals that the striation spacing is 0.5×10^{-6} m at $2a$ = 2.0 cm and 2×10^{-6} m at $2a$ = 4.0 cm crack length. For this material the coefficient C in the crack growth relation of Paris is equal to 4×10^{-13} where da/dn is expressed in m/cycle and ΔK in MPa√m.
    a) What was the applied stress interval $\Delta\sigma$?
    b) How many cycles were needed for crack growth from 2 until 4 cm?

69. For a material 4 constant amplitude test results at 4 different R values (R = –0.1, 0.2, 0.5 and 0.8 respectively) have been found (see the logarithmic and linear plots).
    a) Find the coefficients c_1 and c_2 of the linear crack closure relation $U = c_1 + c_2 \cdot R$. Assume that $\Delta K_{eff} = \Delta K$ for $R = 1$.
    b) What is the crack growth rate da/dn at ΔK_{eff} = 10 and 15 MPa√m?



Figures for exercise 69

70. A cylindrical pressure vessel with a very large diameter has a wall thickness t = 1.25 cm. A half-circular shaped crack of 0.25 cm depth is found on the inner surface. The orientation of the crack is normal to the hoop stress in the cylinder wall. There is a risk of stress corrosion crack growth. For the combination of the material and the environment it is known that $K_{I_{scc}}$ = 10 MPa√m. For safety reasons it is required that, should crack growth occur, the vessel will leak before it breaks.

   a) For which interval of hoop stresses will this apply, *i.e.* both crack growth and leak before break will occur, assuming that for the material K_c = 88 MPa√m?

   b) Answer the same question assuming K_c = 25 MPa√m.

   Hint: Assume that K_I cannot become larger than that for a through-thickness crack with a length of twice the wall thickness.

71. During a J_{Ic} determination using a technique similar to that of Begley and Landes a critical displacement (at the onset of crack growth) is found of 20 mm for an initial crack length of 10 mm. In the figure load displacement curves are given for 3 initial crack lengths (5, 10 and 15

mm). The tests are performed with bend specimens for which the thickness B and the height W are both equal to 20 mm. The measured displacement is that of the load point.

a) How high is J_{Ic}?

b) Approximately determine the value for the critical displacement at an initial crack length of 15 mm.

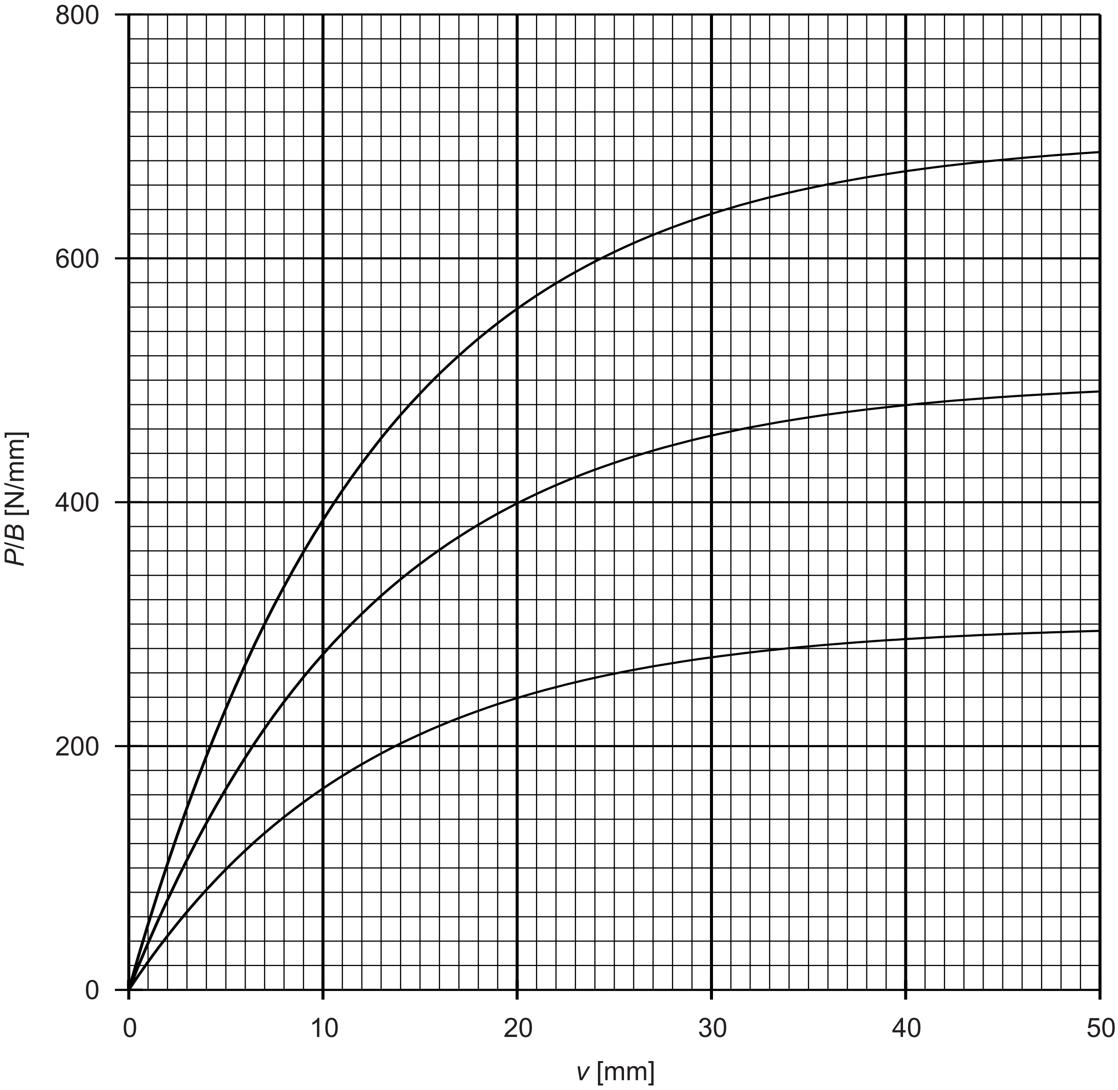


Figure for exercise 71

72. For a certain steel the fatigue crack growth rate da/dn is known as a function of ΔK (see figure). From this steel a cylindrical pressure vessel is made with a diameter of 7.5 meter and a wall thickness of 100 mm. The operational pressure in the vessel is 2 MNm^{-2}. At the design of the vessel it was assumed that failure would occur from a fatigue crack in the axial direction and that leakage would precede failure.
Calculate the number of fatigue cycles (R = 0) between leakage and failure. The K_c value is 200 $MNm^{-3/2}$. Assume that at the onset of leakage a through thickness crack has developed with a length equal to twice the wall thickness.

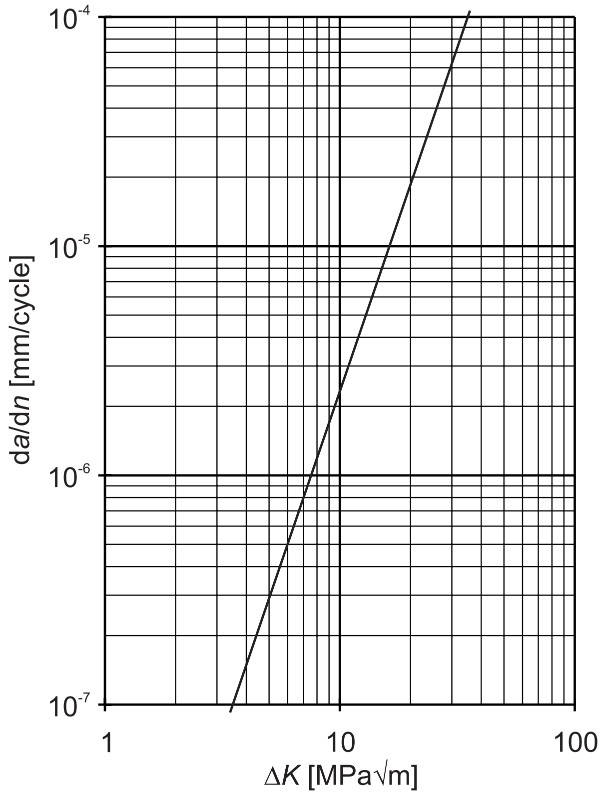


Figure for exercise 72

73. Using a single edge notched bend specimen of nodular cast iron, a J_{Ic} test is performed based on the unloading compliance technique. In figure A the measured load P is plotted as a function of the load displacement v. From these data the slope of the P-v curve during the different unloading-loading cycles are found as follows:

no.	slope [N/mm]	
1	19200	(no crack growth yet)
2	18800	
3	17600	
4	15800	
5	13300	
6	11000	

For specimens with this geometry and made of this material the relative crack length a/W is plotted in figure B as a function of the compliance C. Other data are:

Material: Young's modulus $E = 200\,000$ MPa
yield strength $\sigma_{ys} = 350$ MPa
ultimate tensile strength $\sigma_{uts} = 450$ MPa

Specimen: thickness $B = 7$ mm
height $W = 4 \times B$

a) Roughly indicate the positions of the J-Δa points in figure C, as they follow from the experimental results. Be sure to add numerical values to the vertical axis. J may be calculated without making a distinction between elastic and plastic displacements.

b) Explain how crack tip blunting is quantified for a J_{Ic} test. Indicate this in figure C.

The critical J value according to ASTM E 813 is defined as the value after 0.2 mm of stable crack growth.

c) In figure C, draw the line that represents this stable crack growth.

d) Roughly show in figure C how the critical J-value, J_{Ic}, is obtained.

e) What is the J_{Ic} value (not a calculation but a graphical estimate)?

f) Is this value valid in view of the size requirements?

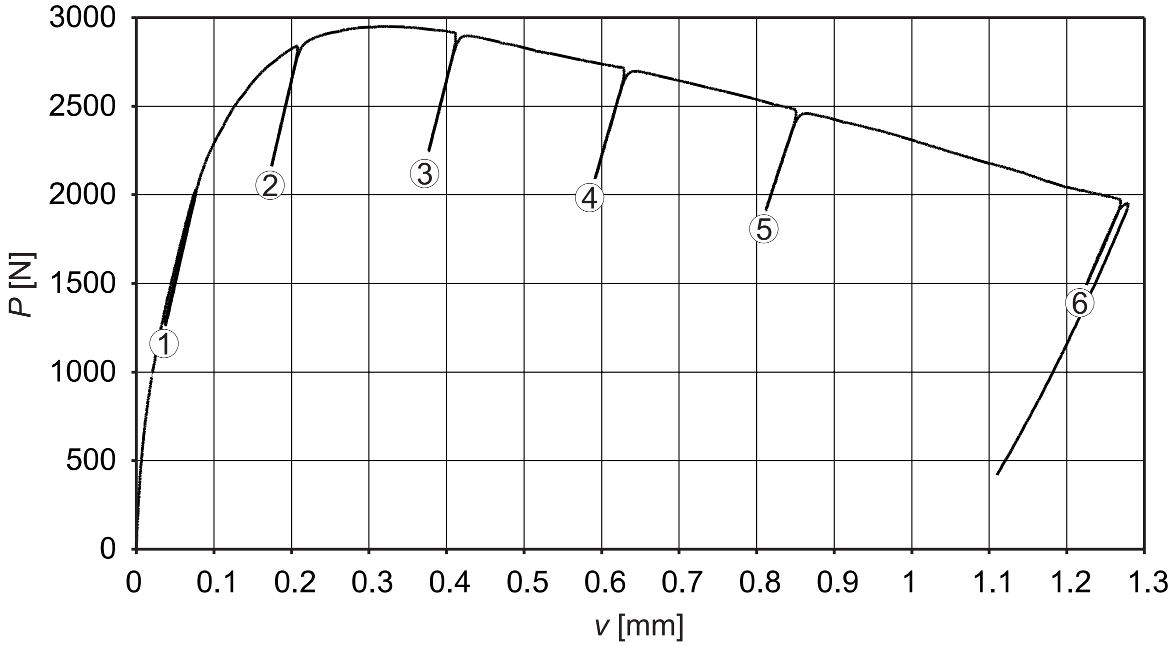


Figure A for exercise 73

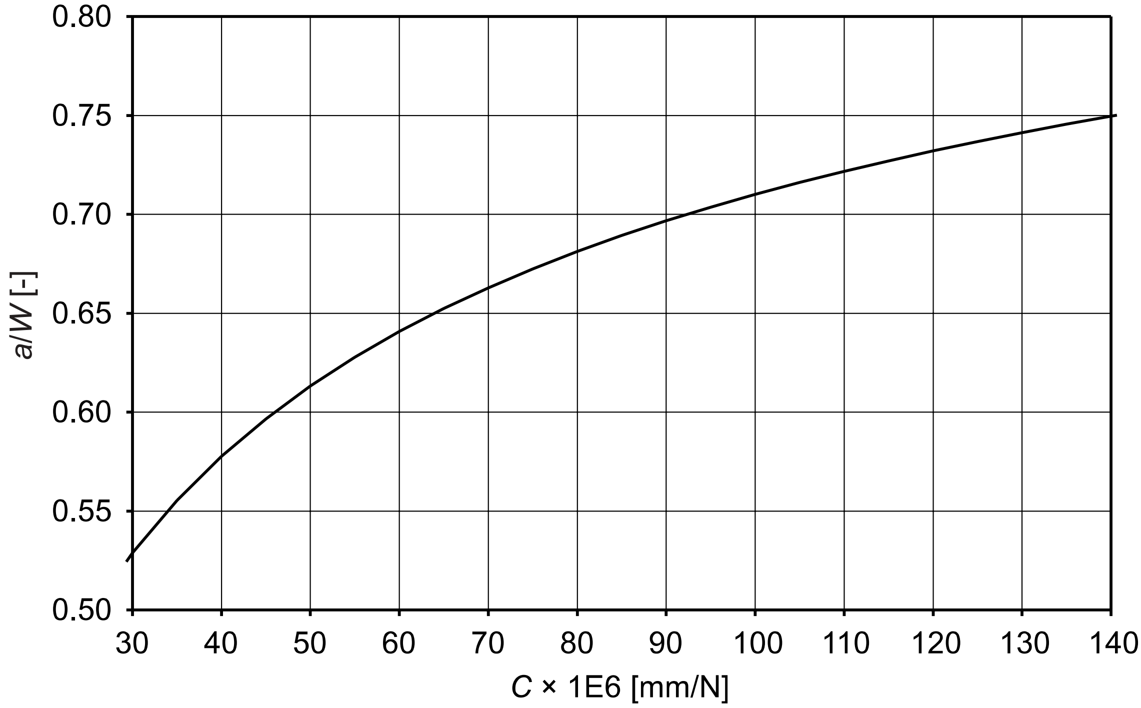


Figure B for exercise 73

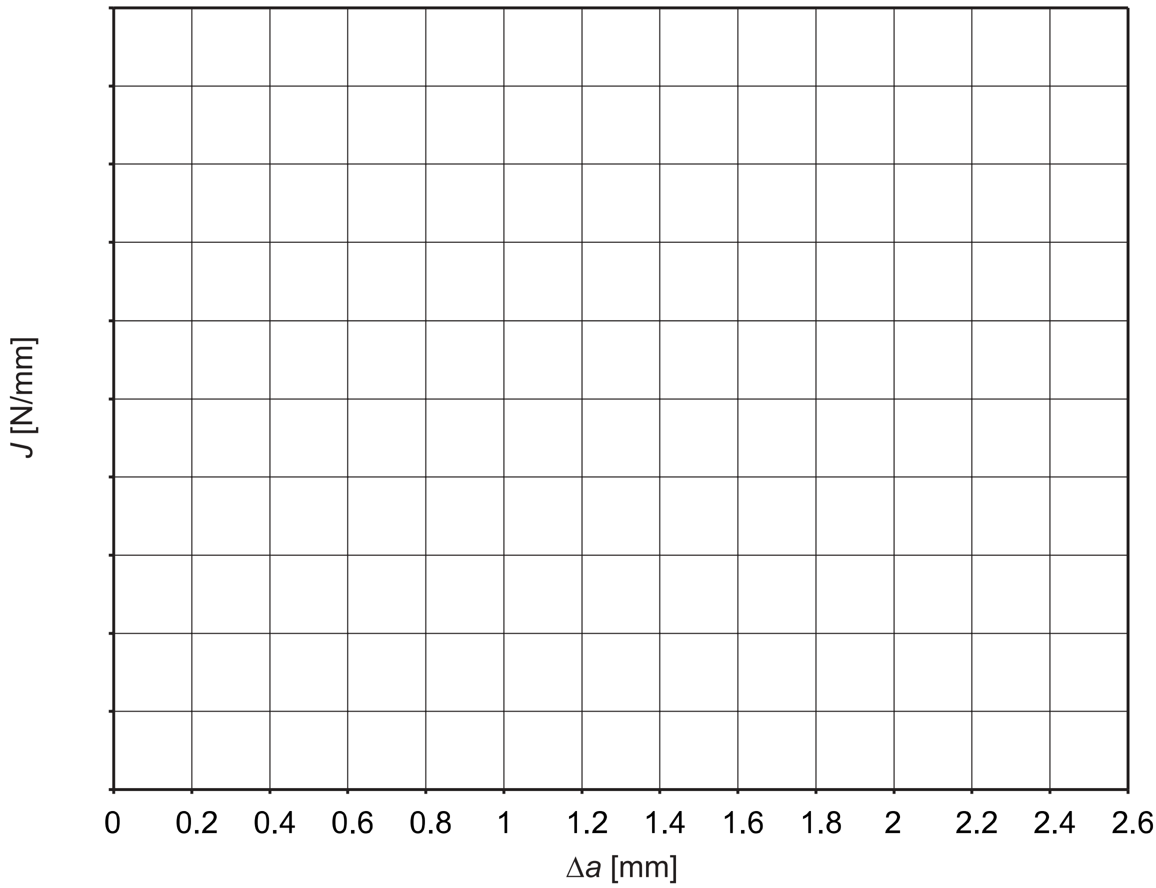


Figure C for exercise 73

74. To determine the susceptibility of a material for stress corrosion cracking, 4 specimens with initial crack lengths of 12.5 mm (specimens with one crack tip) are tested. The times to failure are found as 1, 10, 500 and 5000 hours, while the final crack sizes are 20, 30, 50 and 51 mm respectively.

    Estimate $K_{I_{scc}}$ of the material as accurate as possible.

    Given: specimen width $W = 100$ mm
    plane strain fracture toughness $K_{Ic} = 40$ MPa√m
    geometry factor $f(a/W) = 1.12 + (a/W)^2$

75. Consider a centre cracked plate subjected to a uniform stress of 130 MPa. The fracture toughness $K_c = 50$ MPa√m, the yield strength $\sigma_{ys} = 420$ MPa and the plate width $W = 300$ mm.
    a) What is the maximum allowable crack length?
    b) Calculate the maximum allowable crack length if K_I is corrected for plasticity.

76. In an elastic finite element analysis the stress component σ_y for the first 3 elements in front of the tip of an edge crack are calculated as 170, 98 en 75 MPa respectively. The element size is 2.5 mm and the calculated values may be considered representative for the stress in the centre of each element. In the calculation the crack length is taken as 25 mm, the plate thickness as 10

mm and the plate width as 150 mm.

a) Estimate K_I as accurate as possible.

b) Determine the load applied to the plate.

77. A thin-walled cylindrical pressure vessel is found to contain a serious scratch in the length direction. Reparation by means of welding is less desirable in view of the nature of the contents and will only be done should catastrophic failure occur before the moment of leakage. On the other hand, if the situation remains stable at leakage, the reparation is postponed.
Should reparation take place immediately? Motivate your answer.

Given: vessel diameter = 2500 mm
wall thickness = 25 mm
length of the scratch = 50 mm
maximum pressure = 4 MPa
K_c= 75 MPa√m (at this thickness)

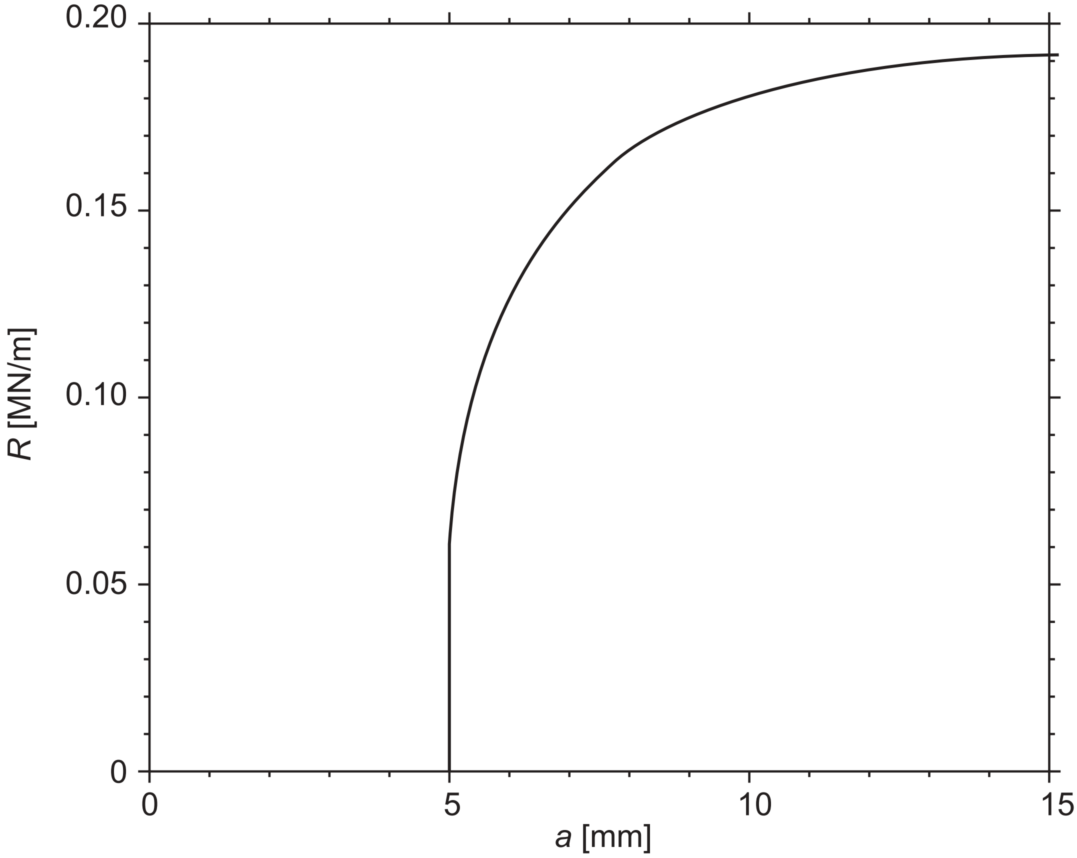


Figure for exercise 78

78. An R-curve is measured for a thin wide plate made of an aluminium alloy containing a central crack with length $2a$ = 10 mm (see figure).

a) Approximately how high is the plane strain fracture toughness K_{Ic} of the aluminium?

b) Determine at which K_I value (K_c) an identical plate will fail if it is loaded normal to the crack.

c) How high is the (nominal) stress in the plate at that moment?

Given: Young's modulus $E = 7 \times 10^4$ MPa
Poisson's ratio = 0.33

79. In a very large steel part a crack is found with a length of $2a$ = 40 mm. The structural part will be applied under circumstances where stress corrosion plays a role. The incubation time for the combination of material, thickness and environment can be derived from the plot in figure A. Information about the stress corrosion crack growth rate is given in figure B.
For the geometry of the part the stress intensity as a function of crack length is given by $K_I = \sigma\sqrt{\pi a}$. The part will be subjected to a constant load resulting in a nominal stress equal to 100 MPa.
What is the lifetime of the part if possible crack growth is allowed as long as $K_I < K_c/4$?

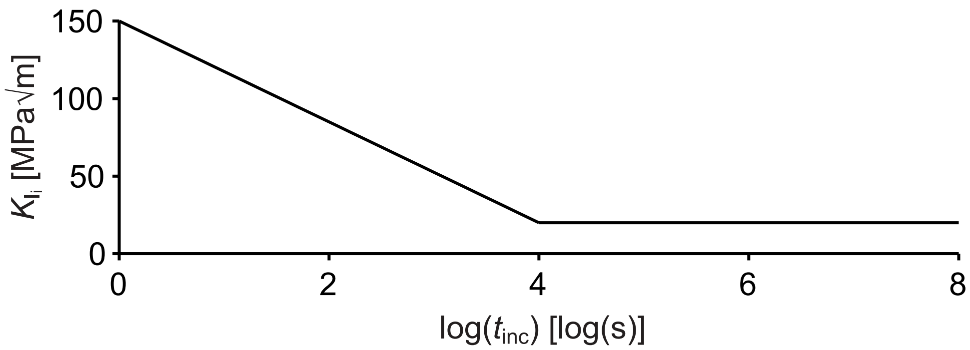


Figure A for exercise 79

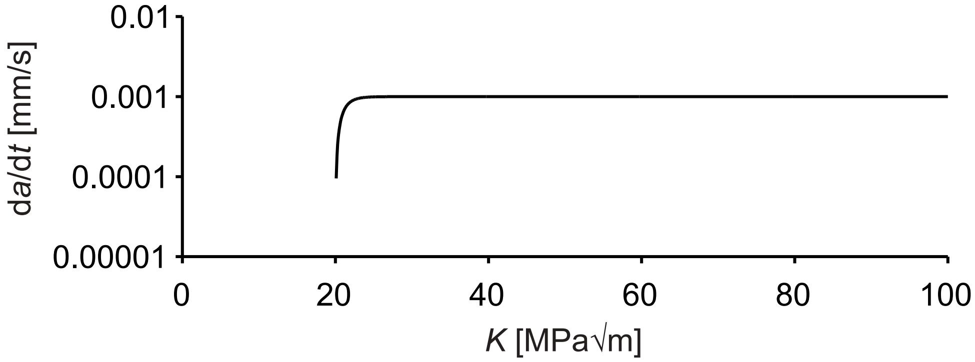


Figure B for exercise 79

80. After being used for a long time a 7 cm long edge crack oriented normal to the load direction was found in a wide panel of some alloy. Investigation showed that crack growth started from a small notch at the edge of the panel. Apparently this notch was present when the panel was put into operation. The panel was designed for constant amplitude fatigue loads (at R = 0) lower than 20 % of the yield strength (σ_{ys} = 345 MPa).

Since the crack length of 7 cm is unacceptably long, it is decided to replace the panel. Examination of the fracture surface at distances of 1.50 and 6.96 cm from the edge reveals striations with average widths of 2.16×10^{-6} and 2.16×10^{-5} m respectively. For the material a crack growth rate relation is known: $da/dn = C(\Delta K_{eff})^m$ with $C = 5.566\times10^{-9}$ and $m = 3$, where da/dn is expressed in m/cycle and ΔK_{eff} is in MPa√m. ΔK_{eff} is defined according to the Elber concept, *i.e.* using $U = 0.5 + 0.4 \cdot R$.

a) The fact that the panel is prematurely taken out of operation, is this due to the notch initially being unnoticed or has there been a fatigue load higher than the above-mentioned 20% of the yield strength?
b) Was the fatigue load of the constant amplitude type?
c) How long would the remaining lifetime (in cycles) have been, if crack growth was allowed until $K_I = K_c/2$?

Given: fracture toughness $K_c = 64.04$ MPa√m
K solution: $K_I = 1.12\, \sigma\sqrt{\pi a}$.

81. The following data are known for a material:
- plane stress fracture toughness $K_c = 35$ MPa√m
- plane strain toughness $K_{Ic} = 25$ MPa√m
- yield stress $\sigma_{ys} = 375$ MPa

Of this material a 25 mm thick double cantilever beam specimen is made, containing only a machined notch. The specimen is subjected to a displacement that is such that cracking occurs.

a) Is it to be expected that the specimen fails? Motivate your answer.
b) How high will K_I eventually become?

One wants to know the threshold value for stress corrosion crack growth in sea water. Literature indicates that this value in any case is lower than 10 MPa√m.

c) How many percent should the displacement be decreased in order to measure this threshold value as quickly as possible?

82. Consider the case described in exercise 15 but now for an applied load σ that varies between 0 and 5 MPa.

a) How high is K_{max}?
b) Immediately after applying the fatigue load, is the nail still attached to the plate?

One has to make the choice whether or not to remove the nail in order to minimise crack growth by the fatigue load.

c) Indicate what the best choice is and motivate this. It may be assumed that fatigue crack growth in polycarbonate does not depend on the loading ratio R.

5 Answers

2. 2×10^{-6} m
3. yes; yes; no
4. 31.5 %
6. a) yes, no and no; b) 0.59, 1.6 and 2.47 mm; c) 756, 1461 and 500 N/mm^2
7. a) 44.3 $N/mm^{3/2}$; b) 35.6 $N/mm^{3/2}$; c) 6.2 N/mm^2
8. a) 6 mm; b) no; c) 3779 $N/mm^{3/2}$; e) $2(R+a) = 64.5$ mm and thus $a = 27.25$ mm
9. a) 2.6 mm; b) $K_c = 2.55$ MPa√m; c) $P_{c,b}/P_{c,a} = 4.4$
10. 171 mm
11. buckling of the crack flanks
12. $P\sqrt{\pi a} + ½\,(F/\sqrt{\pi a} + \sigma\sqrt{\pi a})$
13. a) 41.8 MPa√m; c) 43.8 MPa√m; d) higher, lower; f) $K_c = 53.8$ MPa√m and $K_e = 50.1$ MPa√m
14. a) 9.57 MPa√m; c) 2247 MPa
15. a) 30 $Nmm^{-3/2}$; b) 840 N; c) 30 $Nmm^{-3/2}$; d) yes
17. $K_I = \sigma\sqrt{\pi a}/\sqrt{1-\sigma^2/2\sigma_{ys}^2}$
19. K_I decreases
20. a) 33.6 MPa√m; b) 19.4 mm; c) 110 MPa; d) 30.8 MPa√m
21. too low
24. b) 1905 MPa and 614 MPa√m
25. 64.8 MPa√m
26. material A
27. a) $K_{separate}/K_{together} = 0.65$; b) yes; c) $K_{separate}/K_{together} = 2.09$
28. a) 56 604 N; b) 17 190 N
29. a) Griffith: $\sigma_c = 5.8$ MPa, K_{Ic}: $\sigma_c = 479$ MPa
30. a) 250 N/mm; c) 237 MPa√m; d) 1.1 m
31. $0.85\times\sigma_{ys} = 383$ MPa
32. a) ≈ 260 N/mm; b) ≈ 7700 $N/mm^{3/2}$; c) 643 kg
33. a) 1.055 MN/m; b) yes
35. $B_k \approx 25\,B_j$
42. a) zero
44. a) yes, unlimited; b) no, lifetime 224 cycles
45. case B
46. 18.8×10^{-3} meter
47. repair now (1.31×10^6 cycles to failure)
49. $0.2\times(da/dn)_{ca}$, $0.45\times(da/dn)_{ca}$ and $1\times(da/dn)_{ca}$ where $(da/dn)_{ca} = 1.33\times10^{-6}$ mm/cycle
57. a) nothing; b) yes, there is a risk; c) 131 days

59. 8.98 MPa
61. a) 4.0 mm; b) 4.6 mm; c) yes
62. minimum thickness 4 mm
63. a) 2983 $N/mm^{3/2}$; b) $2.46 < a < 25$ mm; c) $107/4$ days
64. a) 55 MPa√m; b) 219 MPa
65. 4.8 MPa√m
66. 16.7 hours
67. 22 % thicker
68. a) 188.6 MPa ; b) ± 10 000 cycles
69. a) $c_1 = 0.553$, $c_2 = 0.447$; b) 7.7×10^{-4} and 2.2×10^{-3} mm/cycle
70. a) 151 - 444 MPa; b) in case of crack growth the vessel will break before leakage
71. a) 1000 N/mm; b) ≈ 17 mm
72. ≈ 910 000 cycles
73. e) $J_{Ic} \approx 15$ N/mm; f) yes
74. 16.3 MPa√m
75. a) 94 mm; b) 90 mm
76. a) 15 MPa√m; b) 71.7 kN
77. stable situation, do not repair now
78. a) $K_{Ic} \approx 69$ MPa√m; b) $K_c = 100$ MPa√m; c) 690 MPa
79. total time is about 30000 seconds
80. a) due to the notch; b) yes; c) 3267 cycles
81. a) no; b) 25 MPa√m; c) 60%
82. a) 30 $Nmm^{-3/2}$; b) yes; c) do not remove the nail