7 EPFM Testing

7.1 Introduction

In chapter 6 the two most widely known concepts of Elastic-Plastic Fracture Mechanics, the J integral and Crack Opening Displacement (COD) approaches, were discussed in general terms.

This chapter will deal with test methods for obtaining values of J and CTOD, including critical values J_{Ic} and $\delta_{t_{crit}}$. The chapter may be considered the EPFM counterpart of chapter 5, which discussed LEFM test methods.

The greater complexity of the J integral concept as compared to the COD concept is clearly demonstrated by the derivations in chapter 6. This difference in complexity is also found in the test methods. Therefore the discussion of J integral testing is subdivided into three sections:

1) The original J_{Ic} test method, section 7.2.
2) Alternative methods and expressions for J, section 7.3.
3) The standard J_{Ic} test, section 7.4.

The original J_{Ic} test method requires a large amount of data analysis. This problem led to the development of certain types of test specimen for which simple expressions for J could be derived, and ultimately to the standard J_{Ic} test.

Although it is not within the framework of EPFM testing, the K_{Ic} specimen size requirement (see section 5.2) is further discussed in section 7.5. The reason is that the J integral concept enables this criterion to be viewed from a different perspective.

The COD concept is much more straightforward than the J integral, at least from the experimental point of view. Thus only the standard $\delta_{t_{crit}}$ test itself will be described, namely in section 7.6.

With respect to standard test methods, it has already been remarked in sections 6.4 and 6.8 that the J integral concept was developed mainly in the USA and the COD concept in the UK. Consequently it is logical that the original standard for J_{Ic} was American (American Society for Testing and Materials) while the original COD test was the subject of an official British Standard (British Standard Institution, BSI), see references 1 and 2 of the bibliography to this chapter. At present both organizations have incorporated the J integral concept as well as the COD concept into their test standards.

7.2 The Original J_{Ic} Test Method

The first experimental method for determining J (more specifically J_{Ic}, the critical

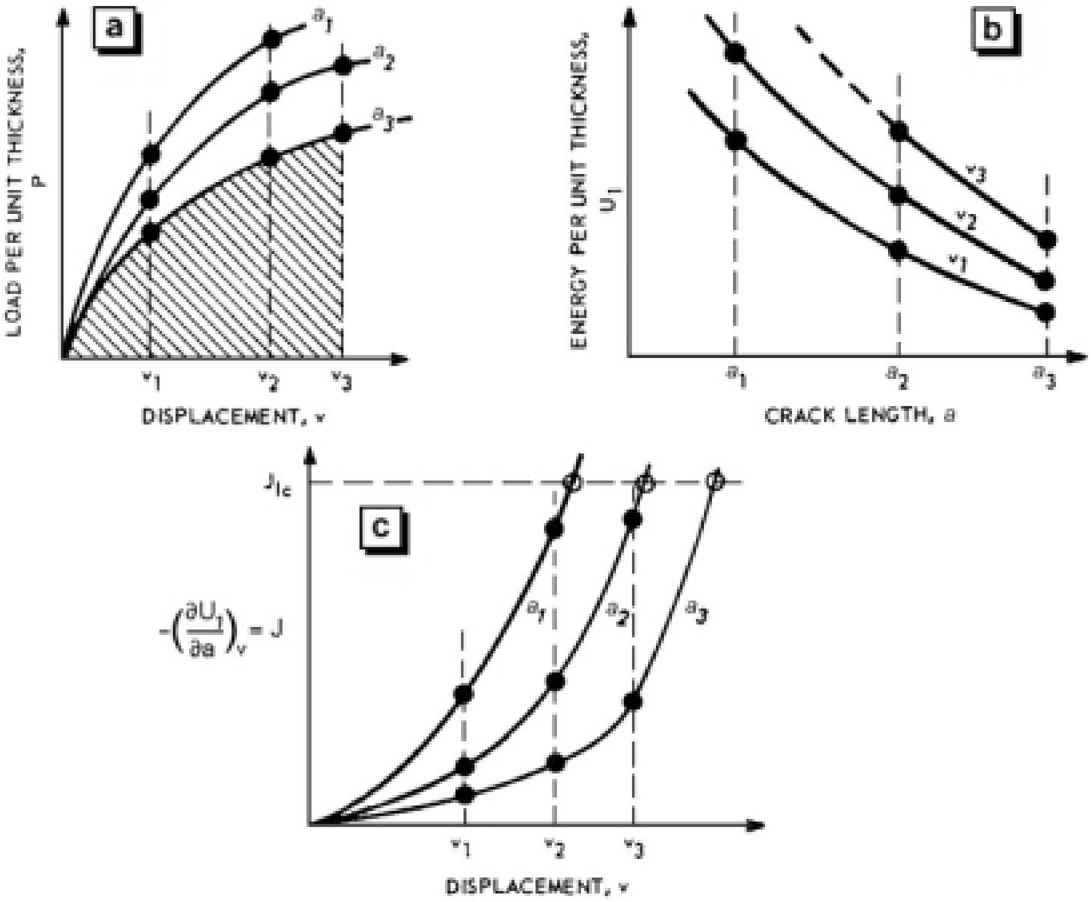

Figure 7.1. The graphical procedure involved in J_{Ic} testing according to Begley and Landes.

mode I value at the onset of crack extension) was published by Begley and Landes in 1971, reference 3 of the bibliography. The method is based on the definition of J as $-\mathrm{d}U_p/\mathrm{d}a$, and requires graphical assessment of $\mathrm{d}U_p/\mathrm{d}a$. The method will be illustrated with the help of figure 7.1, which schematically gives the graphical procedure for obtaining J_{Ic}.

The procedure is as follows:

1) Load-displacement diagrams are obtained for a number of specimens precracked to different crack lengths (a_1, a_2, a_3 in figure 7.1.a). Areas under the load-displacement curves represent the energy per unit thickness, U_1, delivered to the specimens. Thus the shaded area in figure 7.1.a is equal to the energy term U_1 for a specimen with crack length a_3 loaded to a displacement v_3.
2) U_1 is plotted as a function of crack length for several constant values of displacement, figure 7.1.b.
3) The negative slopes of the U_1–a curves, *i.e.* $-\left(\partial U_1/\partial a\right)_v$, are plotted against displacement for any desired crack length between the shortest and longest used in testing, figure 7.1.c. Since the elastic strain energy contents of a specimen is equal to the energy delivered to that specimen, it follows that $-\left(\partial U_1/\partial a\right)_v$ is equal to $-\left(\partial U_a/\partial a\right)_v$. In section 6.3 the energy definition of J was given as:

$$J = -\frac{\mathrm{d}U_\mathrm{p}}{\mathrm{d}a} = \frac{\mathrm{d}}{\mathrm{d}a}(F - U_\mathrm{a}) \, . \tag{6.1}$$

Since for crack extension under fixed grip conditions no work is performed by the loading system, it follows that:

$$J = -\left(\frac{\partial U_\mathrm{p}}{\partial a}\right)_v = -\left(\frac{\partial U_\mathrm{a}}{\partial a}\right)_v \, . \tag{7.1}$$

Hence figure 7.1.c in fact gives J-v curves for particular crack lengths.

4) Knowledge of the displacement v at the onset of crack extension enables J_Ic to be found from the J-v curve for each initial crack length. In figure 7.1.c the value of J_Ic is schematically shown to be constant as, ideally, it should be if J is an appropriate criterion for the onset of crack extension.

Knowledge of the critical displacement v is a weak step in the procedure. Begley and Landes used materials where the maximum in the load-displacement curve characterized the onset of crack growth. For other materials a crack extension measurement device (*e.g.* a potential drop measurement apparatus) is necessary. The method of Begley and Landes has the potential to find the applied J for an unknown geometry.

The graphical procedure described involves a large amount of data manipulation and replotting in order to obtain J-v calibration curves and hence J_Ic. There are thus many possibilities for errors, and so easier methods have been looked for, as will be discussed in section 7.3. However, the elegance of this original test method, in making direct use of the energy definition of J, remains and it is still used as a reference to check more recent developments.

7.3 Alternative Methods and Expressions for *J*

The main contribution to seeking alternatives for the Begley and Landes method was made by Rice *et al.*, reference 4 of the bibliography. Their analysis leads to simple expressions for J for certain types of specimen. However, before these expressions can be discussed it is necessary to consider alternative definitions of J.

Recall the expressions for U_p and J given in equations (4.2) and (6.1) respectively

$$U_\mathrm{p} = U_\mathrm{o} + U_\mathrm{a} - F \tag{4.2}$$

$$J = -\frac{\mathrm{d}U_\mathrm{p}}{\mathrm{d}a} \, . \tag{6.1}$$

We will now consider the value of J for two extreme cases, namely for crack extension under constant displacement v and crack extension under constant load per unit thickness P. It follows that in both cases the change in potential energy due to a crack extension Δa is

$$\Delta U_\mathrm{p} = U_\mathrm{p}\big|_{a+\Delta a} - U_\mathrm{p}\big|_a = \Delta U_\mathrm{a} - \Delta F \, . \tag{7.2}$$

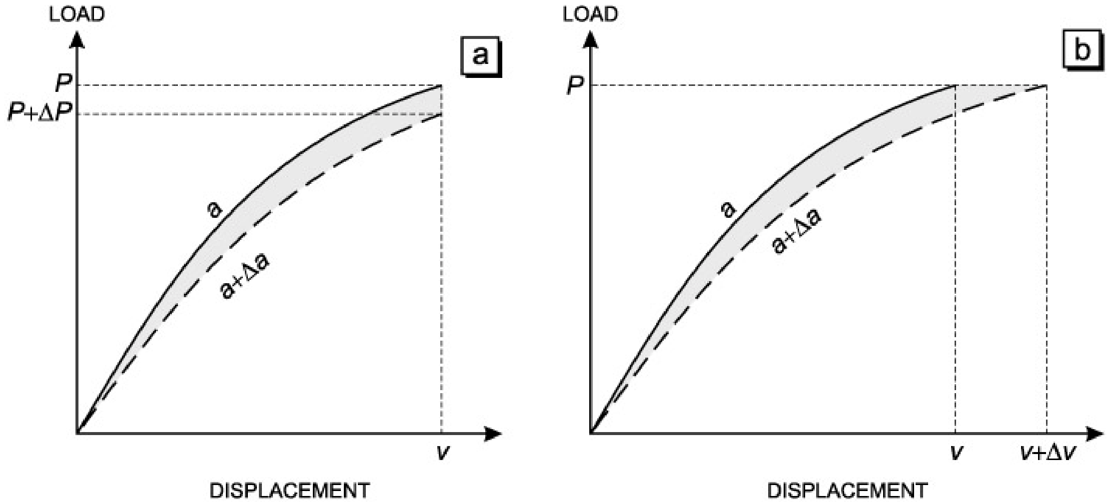

Figure 7.2. Crack extension in a nonlinear elastic body under (a) fixed grip and (b) constant load conditions.

For the case of a fixed grip condition, see figure 7.2.a, we may write

$$\Delta U_{\mathrm{a}} = \int_0^v P\big|_{a+\Delta a}\,\mathrm{d}v - \int_0^v P\big|_{a}\,\mathrm{d}v = \int_0^v \Delta P\,\mathrm{d}v \qquad \text{and} \qquad \Delta F = 0 \tag{7.3}$$

and thus, using equation (7.2), the change in potential energy is equal to

$$\Delta U_{\mathrm{p}} = \Delta U_{\mathrm{a}} - \Delta F = \int_0^v \Delta P\,\mathrm{d}v\,. \tag{7.4}$$

Note that ΔP is negative and that ΔU_{p} is equal to minus the shaded area between the curves for crack lengths a and $a+\Delta a$ in figure 7.2.a. From equation (6.1) it follows that

$$J = -\frac{\mathrm{d}U_{\mathrm{p}}}{\mathrm{d}a} = -\lim_{\Delta a\to 0}\frac{\Delta U_{\mathrm{p}}}{\Delta a} = -\int_0^v \left(\frac{\partial P}{\partial a}\right)_v \mathrm{d}v\,. \tag{7.5}$$

The case of a constant load condition, figure 7.2.b, is slightly more complicated, *i.e.*

$$\Delta U_{\mathrm{a}} = \int_0^{v+\Delta v} P\big|_{a+\Delta a}\,\mathrm{d}v - \int_0^v P\big|_{a}\,\mathrm{d}v \qquad \text{and} \qquad \Delta F = P\,\Delta v\,. \tag{7.6}$$

This leads to

$$\Delta U_{\mathrm{p}} = \Delta U_{\mathrm{a}} - \Delta F = \int_0^{v+\Delta v} P\big|_{a+\Delta a}\,\mathrm{d}v - \left(\int_0^{v} P\big|_{a}\,\mathrm{d}v + P\,\Delta v\right), \tag{7.7}$$

which, when regarded more closely, is equal to the shaded area in figure 7.2.b. Therefore we may rewrite ΔU_{p} as

$$\Delta U_{\mathrm{p}} = -\int_0^{P} \Delta v\,\mathrm{d}P\,. \tag{7.8}$$

Note that Δv is positive now. Thus

$$J = -\frac{\mathrm{d}U_{\mathrm{p}}}{\mathrm{d}a} = -\lim_{\Delta a \to 0}\frac{\Delta U_{\mathrm{p}}}{\Delta a} = \int_0^{P}\left(\frac{\partial v}{\partial a}\right)_{P}\mathrm{d}P\,. \tag{7.9}$$

The same results can also be found purely algebraically. For crack extension under fixed grip conditions J is

$$J = -\left(\frac{\partial U_{\mathrm{p}}}{\partial a}\right)_{v} = \left(\frac{\partial F}{\partial a} - \frac{\partial U_{\mathrm{a}}}{\partial a}\right)_{v} = \left(0 - \frac{\partial U_{\mathrm{a}}}{\partial a}\right)_{v} = -\left(\frac{\partial U_{\mathrm{a}}}{\partial a}\right)_{v} = -\left(\frac{\partial}{\partial a}\int_0^{v} P\,\mathrm{d}v\right)_{v} = -\int_0^{v}\left(\frac{\partial P}{\partial a}\right)_{v}\mathrm{d}v\,,$$

while on the other hand, for the case of constant load conditions

$$J = -\left(\frac{\partial U_{\mathrm{p}}}{\partial a}\right)_{P} = \left(\frac{\partial F}{\partial a} - \frac{\partial U_{\mathrm{a}}}{\partial a}\right)_{P} = \left(P\frac{\partial v}{\partial a} - \frac{\partial}{\partial a}\int_0^{v} P\,\mathrm{d}v\right)_{P} = \left(P\frac{\partial v}{\partial a}\right)_{P} - \frac{\partial}{\partial a}\left(Pv - \int_0^{P} v\,\mathrm{d}P\right)_{P} = \int_0^{P}\left(\frac{\partial v}{\partial a}\right)_{P}\mathrm{d}P\,.$$

Thus the alternative definitions of J, for crack extension under fixed grip or constant load conditions are

$$J = -\int_0^{v}\left(\frac{\partial P}{\partial a}\right)_{v}\mathrm{d}v = \int_0^{P}\left(\frac{\partial v}{\partial a}\right)_{P}\mathrm{d}P \tag{7.10}$$

Note the different sign for fixed grip and constant load conditions. This is analogous to the formulae for G, equations (4.22).

Using equation (7.10), Rice *et al.* showed in 1973 that it is possible to determine J_{Ic} from a single test of certain types of specimen. As an example, J for a deeply cracked bar in bending was derived as

$$J = \frac{2}{Bb}\int_0^{\theta_c} M\,\mathrm{d}\theta_c\,, \qquad (7.11)$$

where B is the thickness of the bar, b is the size of the uncracked ligament ahead of the crack, M is the bending moment and θ_c is the part of the total bending angle θ due to introduction of the crack. More recently, see reference 5 of the bibliography to this chapter, it was found that J is evaluated more accurately by simply using the total bending angle θ instead of θ_c, *i.e.*

$$J = \frac{2}{Bb}\int_0^{\theta} M\,\mathrm{d}\theta \qquad (7.12)$$

Equation (7.12) is important, since it applies to a basic cracked configuration. Therefore a derivation is given in some detail here with the help of figure 7.3. For this deeply cracked bar loaded in bending the ligament size, b, is chosen small compared to the width of the bar, W, so that it may be safely assumed that all plastic deformation is confined to this ligament.

M' is the bending moment per unit thickness, *i.e.* $M' = M/B$. We will use the definition of J for fixed grip conditions, *i.e.* the first form of equation (7.10). P and v are converted to M' and θ by assuming the moment is applied through three-point bending. The load per unit thickness, P, can be written as $4M'/L$, where L is the span of the bend specimen. Furthermore, since plasticity is confined to the ligament, the sides of the beam will remain straight and v is equal to $\theta L/4$. Finally, since $b = W - a$ it follows that $\partial/\partial a = -\partial/\partial b$. The first form of equation (7.10) can now be written as

$$J = -\int_0^{v}\left(\frac{\partial P}{\partial a}\right)_v \mathrm{d}v = -\int_0^{\theta}\left(\frac{\partial M'}{\partial a}\right)_\theta \mathrm{d}\theta = +\int_0^{\theta}\left(\frac{\partial M'}{\partial b}\right)_\theta \mathrm{d}\theta\,. \qquad (7.13)$$

Since this expression cannot be evaluated experimentally, an analytical relation must be found between θ, b and M'. Rice *et al.* argued that a dimensional analysis can be used to obtain this relation. However, here an alternative reasoning will be used.

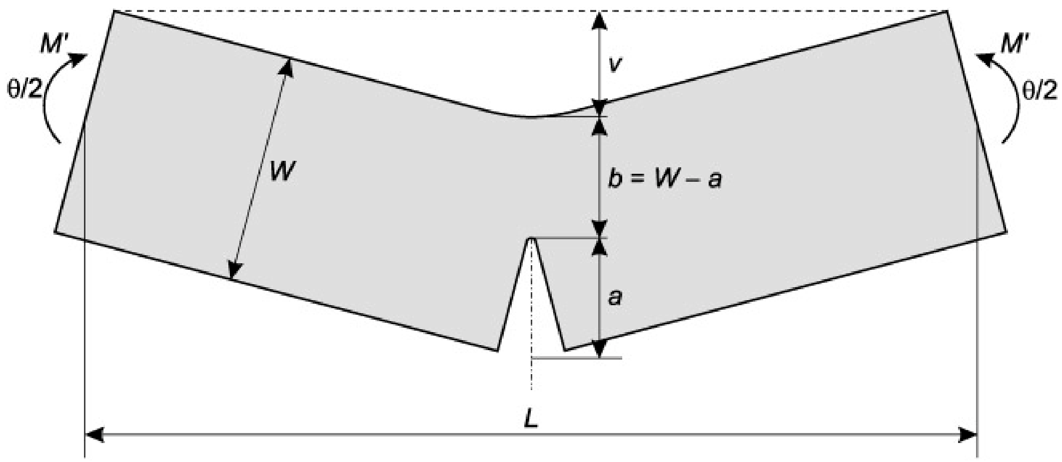

Figure 7.3. A deeply cracked bar loaded in bending.

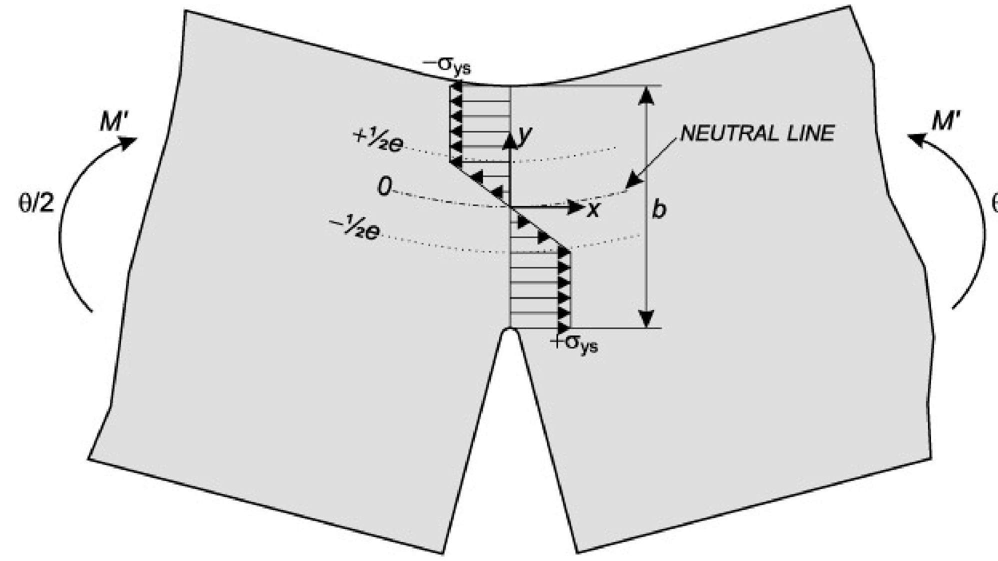

Figure 7.4. Stress distribution in the critical ligament

Elastic – perfectly plastic material behaviour is assumed, leading to a distribution in the ligament of the stress component parallel to the neutral line, σ_x, as follows (see figure 7.4):

$$\sigma_x(y) = \begin{cases} \dfrac{2y}{e}\sigma_{ys} & \text{for } |y| < \tfrac{1}{2}e \\ \sigma_{ys} & \text{for } \tfrac{1}{2}e < |y| < \tfrac{1}{2}b\ , \end{cases} \tag{7.14}$$

where y is the distance from the neutral line. The moment corresponding to this stress distribution can be straightforwardly calculated as

$$M' = \int_{-\frac{1}{2}b}^{+\frac{1}{2}b} y\,\sigma_x(y)\,\mathrm{d}y = \sigma_{ys}\left(\frac{b^2}{4} - \frac{e^2}{12}\right). \tag{7.15}$$

The width e of the elastic part of the ligament is now estimated by assuming that in the small region around the ligament the neutral line takes the shape of a circle segment with radius R and that all planes normal to the neutral line remain normal to that line. Under these assumptions the strain, ε_x, parallel to the neutral line can be written as a function of y:

$$\varepsilon_x(y) = \frac{2\pi(R-y) - 2\pi R}{2\pi R} = -\frac{y}{R}\,. \tag{7.16}$$

At the boundary between the elastic and plastic parts of the ligament, *i.e.* $y = \pm\tfrac{1}{2}e$, the absolute value of ε_x is approximately equal to the yield stress divided by E', *i.e.* E for plane stress and $E/(1-\nu^2)$ for plane strain. Thus using equation (7.16) it follows that

$$\frac{\sigma_{ys}}{E'} \approx \frac{\tfrac{1}{2}e}{R} \quad \text{or} \quad e \approx \frac{2R\sigma_{ys}}{E'}\,.$$

Now it is assumed that the length of the circular shaped segment of the neutral line in the region around the ligament is roughly of the order of the size of the ligament b, *i.e.* $R\theta \approx b$. This leads to

$$e \approx \frac{2b\sigma_{ys}}{\theta E'}\,. \tag{7.17}$$

Substitution of this expression in equation (7.15) leads to

$$M' = \frac{b^2\sigma_{ys}}{4}\left\{1 - \frac{4}{3}\left(\frac{\sigma_{ys}}{\theta E'}\right)^2\right\} . \tag{7.18}$$

The significance of this equation is that

$$M' = b^2 F(\theta) , \tag{7.19}$$

where the function $F(\theta)$ depends on material properties such as E, ν, σ_{ys}, and, in the case of a work hardening material, on the work hardening exponent n. We may now write

$$\left(\frac{\partial M'}{\partial b}\right)_\theta = 2b\,F(\theta) = 2\frac{M'}{b} . \tag{7.20}$$

Substituting this expression in equation (7.13), we find

$$J = \int_0^\theta 2\frac{M'}{b}\,d\theta = \frac{2}{b}\int_0^\theta M'\,d\theta = \frac{2}{B(W-a)}\int_0^\theta M\,d\theta . \tag{7.21}$$

For a deeply cracked bar it is reasonable to assume that all plasticity is restricted to the ligament, and thus the two halves of the bar remain straight. This enables equation (7.12) to be rewritten as the more practical expression

$$J = \frac{2}{Bb}\int_0^v P\,dv , \tag{7.22}$$

where P is the load in terms of a force, *i.e.* no longer defined per unit thickness, and v is the displacement in the load line, termed the *load-line displacement*.

In a J_{Ic} test the load P acting on a cracked bar is measured as a function of the load-line displacement v. Using equation (7.22) J can then be determined for any displacement by calculating the area under the P-v curve up to that displacement, U. At the onset of crack extension, J is equal to J_{Ic}. Therefore

$$J = \frac{2U}{Bb} \quad \text{and} \quad J_{Ic} = \frac{2U_{cr}}{Bb} . \tag{7.23}$$

where U_{cr} is the area under the P-v curve at the onset of crack extension.

Hence in principle J_{Ic} can be determined by performing one test only in which the specimen is loaded until the onset of crack extension. However this is not normally done. The reason is that detection of the beginning of crack extension is difficult. It can only be done with costly apparatus as potential drop, acoustic emission, ultrasonic, eddy current etc., where each has its specific difficulties. An alternative is to make a number of tests whereby each specimen is loaded to give a small but different crack extension Δa. Then the values of J (which are, strictly speaking, invalid) are plotted versus Δa and extrapolated to $\Delta a = 0$ in order to obtain J_{Ic}. An example of this method is given in figure 7.5.

J-Δa lines like those in figure 7.5 are called J resistance curves, by analogy with the

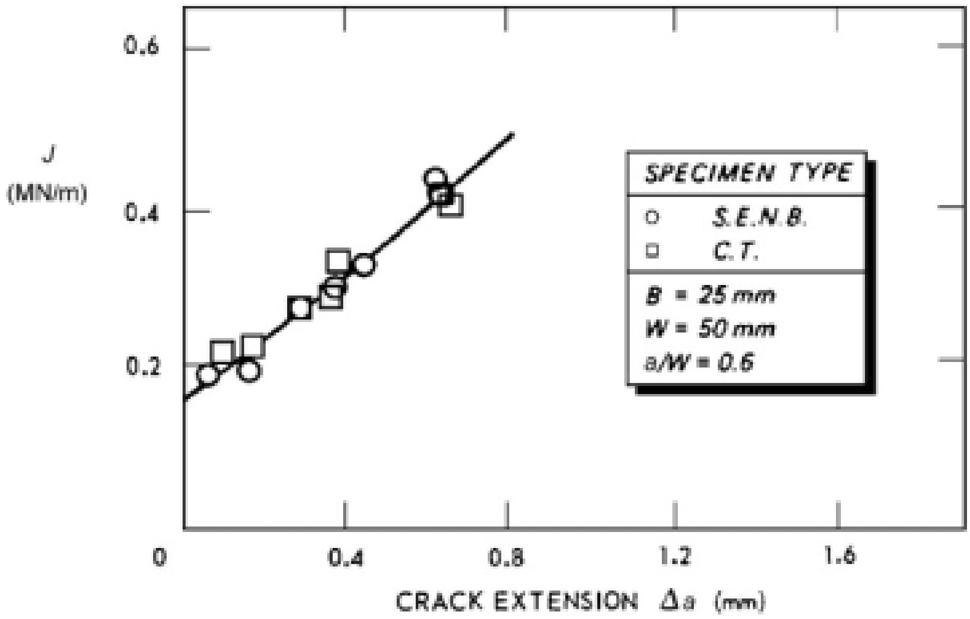

Figure 7.5. J-Δa plots for A-533B steel, after reference 6 of the bibliography.

LEFM *R*-curve. This seems slightly misleading, since J is strictly valid only up to the beginning of crack extension and not beyond it. However, it must be noted that under certain restrictions J resistance curves can be used to predict stable crack extension. This subject is discussed in chapter 8.

The J integral expression in equation (7.23) and the multiple specimen method just described, form the basis for the standard J_{Ic} test, which is discussed in the next section.

7.4 The Standard J_{Ic} Test

Before publication of the standard J_{Ic} test some ten different procedures had been used. Chipperfield (reference 7) reviewed these methods and showed that J_{Ic} values obtained in different ways varied by up to 20%. This clearly demonstrated the need for a standard test.

Original J_{Ic} Test Standard

A proposal for a standard J_{Ic} test was published in 1979. This proposal became an ASTM standard and was first published as such in 1981 under the designation ASTM E 813, reference 1 of the bibliography. This standard describes J_{Ic} determination using three-point notched bend (SENB) and compact tension (CT) specimens. Roughly these are the same specimen geometries as those for K_{Ic} testing (see figures 5.2 and 5.3), but there are a number of differences in detail. For both specimen configurations J is given simply by a form of equation (7.23), *i.e.* $J = (2U/Bb)\cdot f(a/W)$, where $f(a/W)$ depends on the specimen type.

Revised Test Standard

In 1989 a revised version of standard E 813 was published, which is referenced as number 8 of the bibliography. In this standard the same specimen geometries are described, but for experimental reasons J is evaluated in a somewhat different way. The load-line displacement is divided into an elastic and a plastic part, *i.e.* $v = v_{el} + v_{pl}$. Consequently, reverting to equation (7.10), with P now no longer defined per unit thickness, we may write

$$J = \frac{1}{B}\int_0^P \left(\frac{\partial v}{\partial a}\right)_P \mathrm{d}P = \frac{1}{B}\int_0^P \left(\frac{\partial v_{el}}{\partial a}\right)_P \mathrm{d}P + \frac{1}{B}\int_0^P \left(\frac{\partial v_{pl}}{\partial a}\right)_P \mathrm{d}P = J_{el} + J_{pl}\,. \tag{7.24}$$

Expressing v_{el} in terms of the specimen compliance, *i.e.* $v_{el} = C{\cdot}P$, it follows that (*cf.* equations (4.15.b) and (4.18))

$$J_{el} = \frac{1}{B}\int_0^P \left(\frac{\partial v_{el}}{\partial a}\right)_P \mathrm{d}P = \frac{1}{B}\int_0^P \left(\frac{\partial (C{\cdot}P)}{\partial a}\right)_P \mathrm{d}P = \frac{P^2}{2B}\frac{\partial C}{\partial a} = G = \frac{1-\nu^2}{E}K_I^2\,. \tag{7.25}$$

Since the SENB and CT specimens have the same geometry as the standard K_{Ic} specimens, K_I can be calculated using equations (5.1) and (5.2).

Using the same reasoning as given in the previous section, the plastic part of J, J_{pl}, can be related to the area under the P-v_{pl} curve up to the current value of v_{pl}, U_{pl}. The ASTM standard uses the relation

$$J_{pl} = \frac{\eta U_{pl}}{B_N b}\,, \tag{7.26}$$

where η = plastic work factor = $\begin{cases} 2 & \text{for SENB specimens} \\ 2 + 0.522\, b/W & \text{for CT specimens} \end{cases}$

B_N = net specimen thickness, which is equal to B if no side grooves are present.

Figure 7.6 illustrates how the plastic work U_{pl} is calculated. First the total work U is determined by integrating the P-v curve and then the elastic part of the work is subtracted. This elastic part is equal to $\frac{1}{2}v_{el}P$ or, using the elastic specimen compliance C, equal to $\frac{1}{2}CP^2$.

Clearly C has to be known to carry out this procedure. Note also that C depends on the current crack length. It can be determined either by calculating it using the formulae given in the ASTM standard that express C as a function of crack length, specimen dimensions and Young's modulus or by measuring it directly through partial unloading during the test (see also under the next subheading).

J_{Ic} Test Procedure

The steps involved in setting up and conducting a J_{Ic} test are:

1) Selection of specimen type (notch bend or compact tension) and preparation of shop drawings.

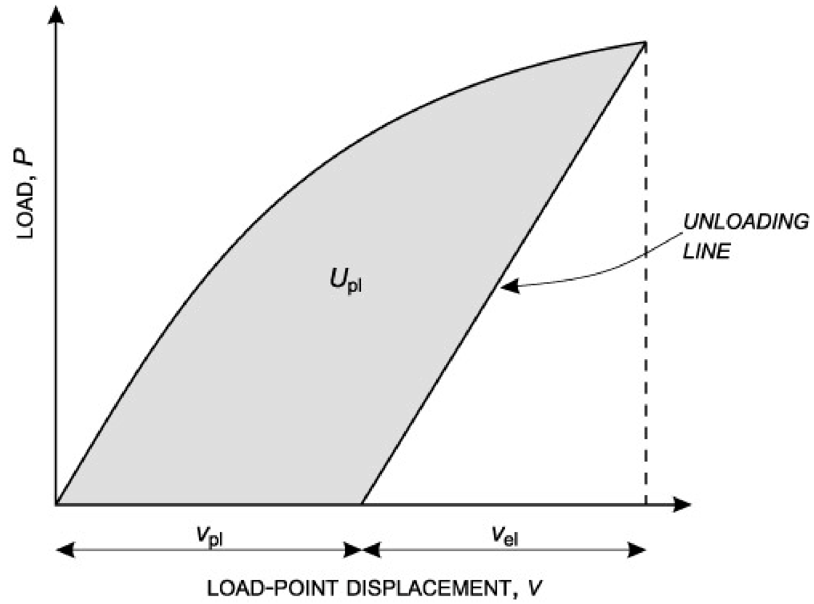

Figure 7.6 The part of the area under the P-v curve that represents the plastic work U_{pl}.

2) Specimen manufacture.
3) Fatigue precracking.
4) Obtain test fixtures and clip gauge for crack opening displacement measurement.
5) Testing.
6) Data analysis.
7) Determination of a provisional J_{Ic} (J_Q).
8) Final check for J_{Ic} validity.

Steps (1) – (5) will be concisely reviewed here insofar as they differ from similar steps for K_{Ic} testing in section 5.2. Steps (6) – (8) are considered under the next sub-heading in this section.

For both SENB and CT specimens the initial crack length (*i.e.* notch plus fatigue precrack) must be greater than 0.5 W to ensure validity of the formulae used to evaluate J. The maximum crack length is 0.75 W, while a value of 0.6 W is usually optimum from an experimental viewpoint.

A special feature of J_{Ic} testing is that the clip gauge has to be positioned in the load line. For the CT specimen this means that the shape of the starter notch is different to

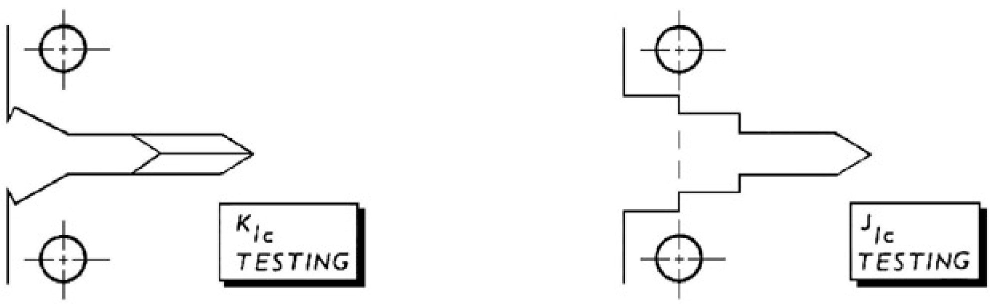

Figure 7.7. CT specimen starter notches.

that used in K_{Ic} testing, figure 7.7. Note that a chevron starter notch for J_{Ic} testing is not specifically recommended. This is also true for the SENB specimen. Experience has shown that a straight starter notch is usually sufficient.

In order to obtain sufficiently sharp crack tips the specimen should be fatigue pre-cracked with the maximum load not exceeding 40% of the limit load for plastic collapse P_L, which can be calculated from

$$\text{SENB specimen} \quad P_L = \frac{4B(W-a)^2\sigma_o}{3S} \tag{7.27}$$

$$\text{CT specimen} \quad P_L = \frac{B(W-a)^2\sigma_o}{(2W+a)}, \tag{7.28}$$

where σ_o is called the flow stress and is typically the average of the yield strength σ_{ys} and the ultimate tensile strength σ_{uts}, *i.e.* $\sigma_o = \frac{1}{2}(\sigma_{ys} + \sigma_{uts})$. The use of σ_o is to account for strain hardening.

The J_{Ic} tests must be carried out under controlled displacement conditions in order to obtain stable crack extension over the whole test range. This means that preferably an electro-mechanical testing machine must be used.

In section 7.3 it was stated that the basis for the standard J_{Ic} test is the multiple specimen method, *i.e.* a number of specimens are loaded to give small but different amounts of crack extension Δa. However, ASTM E 813 does allow a truly single specimen J_{Ic} determination. This involves the use of some technique for measuring the current crack extension during a test, enabling the determination of the J resistance curve defined in section 7.3.

A frequently used method for crack-length monitoring is the *unloading compliance technique*. After loading the specimen until a small amount of crack extension occurs,

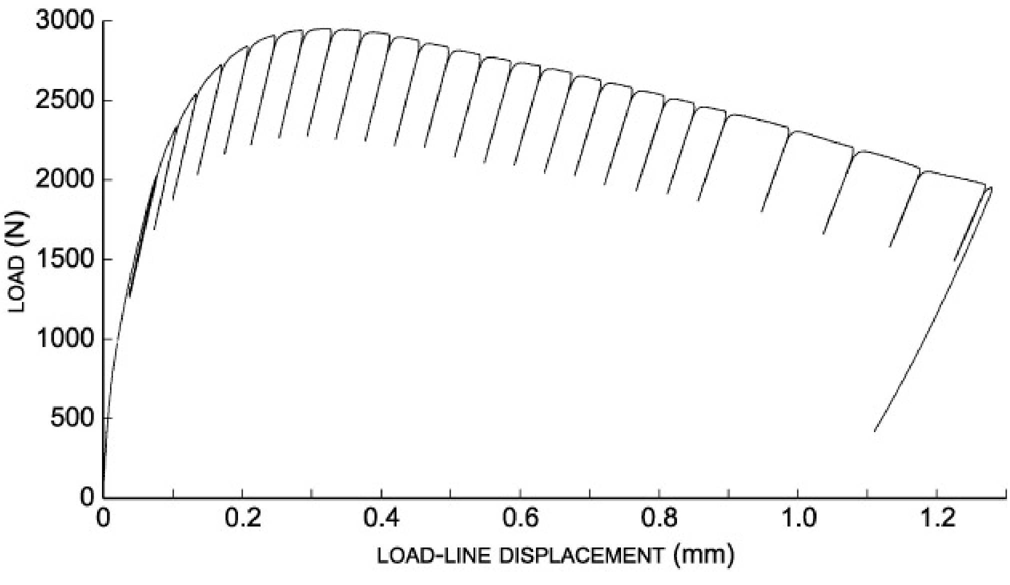

Figure 7.8. An example of the unloading compliance technique.

the load is partially removed and subsequently reapplied, see figure 7.8. To avoid reversed plasticity having any effect on the test results, the maximum unloading range is set to the smaller of 50% of the current load or 20% of P_L. In the load-displacement (P–v) diagram this loading procedure is reflected as the first part of the elastic compliance line for unloading. From the resulting elastic compliance, C, the instantaneous crack length, a, and thus also $\Delta a = a - a_o$ can be calculated. ASTM E 813 gives formulae equating the dimensionless crack length a/W to the dimensionless compliance for SENB and CT specimens. The current values for a, Δa, P and the P-v curve up to the current displacement lead to one point on the J–Δa curve. By repeating this process a number of times a J resistance curve can be obtained from a single specimen. A disadvantage of the method is that an accurate measurement of the unloading compliance line requires suitable equipment and sufficient experimental skill.

For both the multiple and the single specimen technique the specimen is broken afterwards to measure the crack extension visually from the crack surface. Note that for the single specimen technique this final crack extension is determined only to verify the accuracy of the unloading compliance technique. To be able to measure the crack extension a marking technique must be employed for distinguishing between Δa and the residual fracture due to breaking open the specimen after testing. One possibility is heat tinting, *i.e.* heating the specimen in air to cause oxide discoloration of existing crack surfaces. Another is to fatigue cycle after the J_{Ic} test. Details of these techniques are given in reference 8 of the bibliography.

The measurement of the crack extension gives specific problems. J integral test specimens are usually thick, such that ‘crack front tunnelling’ occurs during both precracking and J_{Ic} testing. This is illustrated schematically in figure 7.9. Experience has shown that to obtain consistent values of J and J_{Ic} it is necessary to take averages of at least nine measurements of a and Δa equally spaced across the specimen thickness, and to count the averages of side surface crack lengths as one measurement only.

Data Analysis and Determination of J_{Ic}

The data analysis consists of calculating J values for a number of crack extensions Δa.

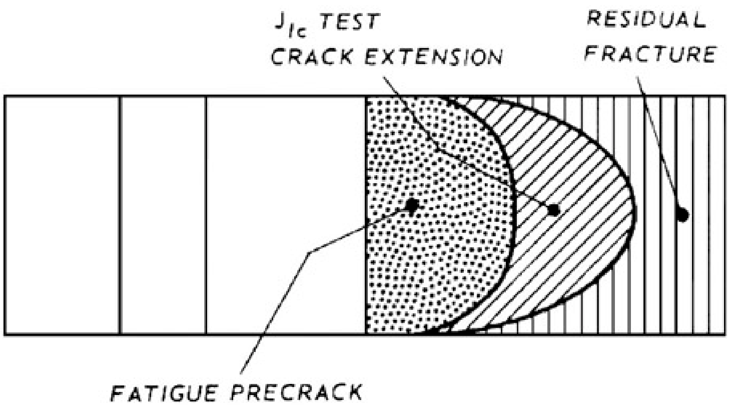

Figure 7.9. Schematic of a J_{Ic} test specimen broken open after testing.

The elastic part of each J value, J_{el}, is evaluated with equation (7.25) by substituting the K_I value corresponding to the load and the crack length at the moment the crack extension Δa was reached. For J_{pl} the load-displacement record is analysed to obtain the area U_{pl} under the curve up to the P-v point corresponding to crack extension Δa. Values of J_{pl} are then calculated by inserting U_{pl} and values of the crack length a into equation (7.26).

These J-Δa points are used in determining the provisional J_{Ic} (J_Q). However, depending on their value, some points may yet turn out to be unacceptable. To check for acceptability and at the same time determine J_Q a plot more or less similar to figure 7.5 must be constructed as shown schematically in figure 7.10.

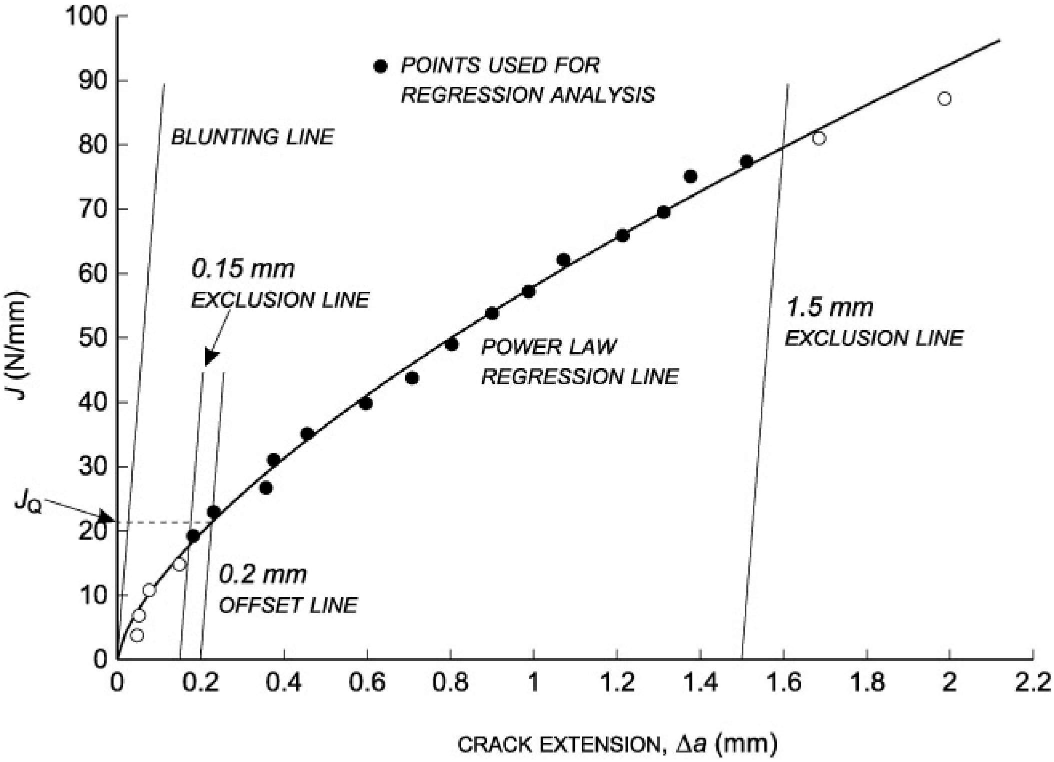

Figure 7.10. Schematic determination of acceptable J values and J_Q.

The procedure for constructing this figure is:

1) Plot the J-Δa data points, discarding points with J values exceeding $b\sigma_o/15$.
2) Draw a theoretical blunting line $J = 2\sigma_o\Delta a$.
3) Draw a 0.2 mm offset line parallel to the blunting line.
4) Draw 0.15 and 1.5 mm exclusion lines parallel to the blunting line and discard all J-Δa points that fall outside the region bounded by these lines.
5) There must be at least 4 J-Δa data points remaining and they must be distributed sufficiently even within the region between the exclusion lines (see reference 8).
6) Using the acceptable J-Δa points, draw a power law regression line of the form $J = C_1(\Delta a)^{C_2}$ by determining a least squares linear regression relation according to:

$$\ln J = \ln C_1 + C_2 \ln (\Delta a) \,. \qquad (7.29)$$

7) Determine the intersection of the power law regression line with the 0.2 mm offset line. The resulting J value is designated J_Q. The ASTM standard suggests an iterative procedure to determine the point of intersection with sufficient accuracy.
8) Draw two vertical lines through the intersections of the exclusion lines with the regression line. These vertical lines represent the minimum and maximum crack extensions. If data points fall outside this range they should be discarded and the procedure should be repeated starting at point 5.

Finally, for J_Q to qualify as a valid J_{Ic}, it is required that:

1) The specimen dimensions satisfy the equation

$$B \text{ and } W - a \text{ both} > \frac{25\, J_Q}{\sigma_o}\,. \tag{7.30}$$

2) The slope of the regression line at J_Q is smaller than σ_o.
3) None of the test specimens have experienced brittle fracture.
4) No excessive crack front tunnelling has occurred (see reference 8).
5) For the single specimen technique the predicted final crack extension does not deviate more than 15% from the crack extension measured directly from the crack surface.

Some Background to the J_{Ic} Determination

1) The minimum thickness requirement $B > 25\ J_Q/\sigma_o$ ensures that crack extension Δa occurs under plane strain. It is an empirical requirement based on tests with steels.
2) The minimum ligament length requirement $b = (W - a) > 25\ J_Q/\sigma_o$ is also empirical and is intended to prevent net section yield, see section 6.1. For this same reason all measured J values exceeding $b\sigma_o/15$ are discarded.
3) The blunting line procedure was adopted to account for the apparent increase in crack length owing to crack tip blunting. This apparent increase in crack length will be less than or equal to the blunted crack tip radius, which in turn is half the crack opening displacement δ_t. Thus the apparent $\Delta a \leq 0.5\delta_t$. Assuming $\delta_t = J/\sigma_o$, a relation discussed earlier in section 6.8, the apparent crack extension due to crack blunting can be accounted for by $\Delta a = 0.5\delta_t = J/2\sigma_o$, or

$$J = 2\sigma_o \Delta a\,. \tag{7.31}$$

Although the concept of accounting for crack blunting is correct, the use of equation (7.31) can still be criticised for two reasons, which are discussed in points 5 and 6.

4) J_Q is not the J value at the initiation of crack extension, since it is determined as the intersection of the 0.2 mm offset line and the power law regression line. In the original ASTM standard (see reference 1) J_Q was determined as the intersection of a linear regression line and the blunting line and could thus be regarded as J at initiation. This procedure, however, was found to introduce much scatter in J_{Ic} values, because the transition between the blunting process and actual crack extension is not always

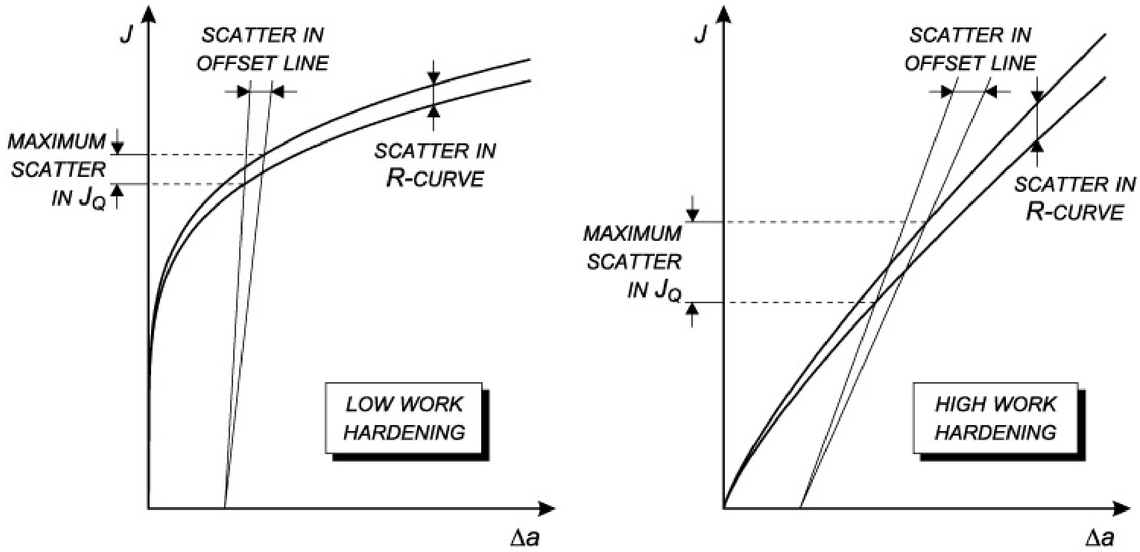

Figure 7.11. Influence of work hardening on J_Q estimation error.

distinct.

Note that the current approach is analogous to that for quantities like the yield strength defined at 0.2% offset strain and K_{Ic} defined at 2% stable crack growth.

5) The blunting line and the J resistance curve are influenced by work hardening. With more work hardening the slope of the blunting line is less, while the J resistance curve is observed to be steeper. This leads to much more potential error in estimating J_Q, as is shown in figure 7.11. In the ASTM procedure this point is addressed by the requirement that the slope of the regression line at J_Q is smaller than σ_o.
6) The blunting line, equation (7.31), is based on $J = \delta_t\sigma_o$. As was discussed in more detail in chapter 6, relations of the form $J = M\delta_t\sigma_o$ are reasonable, but the factor M can vary between 1 and 3, and often has a value ~2. This means that the blunting line slope according to the ASTM standard may be too shallow, which results in an over-

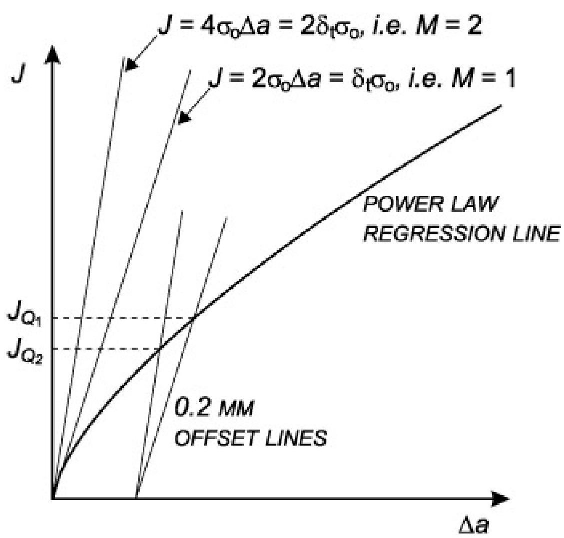

Figure 7.12. Influence of the relation between J and δ_t on J_Q.

estimation of J_Q, as figure 7.12 shows. Experiments have shown that the overestimation of J_Q may be as much as 10%, reference 7 of the bibliography.

It should be noted that in recent ASTM publications (*e.g.* reference 10) the use of a higher blunting line slope, obtained from experimental data, is suggested.

7) The 0.15 mm exclusion line ensures that Δa is at least 0.15 mm and so can be measured accurately enough. The 1.5 mm exclusion line ensures that Δa is generally less than 6% of the remaining ligament in the SENB and CT specimens proposed for J_{Ic} testing, and it has been shown that up to this amount of crack extension the J integral formula, equation (7.26), remains valid.

8) Steps 5 and 8 of the procedure to construct figure 7.10 and the final checking criteria 4 and 5 for J_{Ic} validity have been devised to minimise scatter and improve the reliability of the J resistance curve.

Concluding Remarks

It should be noted that the test procedure according to ASTM standard E 813 allows only J_{Ic} (or J_Q) to be determined. There are also standardized test procedures for determining the whole J resistance curve, involving larger amounts of stable crack extension than for the J_{Ic} determination. With the resulting curve the effect of stable crack growth on the material's crack resistance in the elastic-plastic regime is quantified. This type of test will not be discussed here, but the topics of J controlled crack growth and use of the J resistance curve will be elaborated on in chapter 8.

The J_{Ic} test procedure described in this section is restricted to cases of crack extension by means of a ductile failure mechanism (see chapter 12). However, J can also be used to characterize the onset of brittle fracture, before or during stable crack extension. The restrictions imposed on the amount of crack tip constraint are then much more severe (see reference 9).

It should be further noted that in 1997 the ASTM published a standard (see reference 10) that combines different types of fracture toughness measurements into a single set of test rules. It includes the determination of K_{Ic}, J_{Ic}, J resistance curve, $\delta_{t_{crit}}$ (see section 7.6) and also critical values for J and δ_t in the case of brittle fracture. The idea behind this new standard is to enable fracture toughness evaluation using a single experimental procedure, while minimising the risk of invalid test results because of unexpected material behaviour. If the evaluation of one critical fracture parameter fails it may be possible to evaluate another parameter using the same experimental data. However, the procedure for determining J_{Ic} described in this section 7.4 is more or less copied in this recent ASTM standard, and is therefore still relevant within the context of this course.

7.5 The K_{Ic} Specimen Size Requirement

Although it is not really part of EPFM testing, some attention will be paid to the evaluation of K_{Ic} for relatively tough materials. The reason is that J resistance curves enable a somewhat different view on this subject.

K_{Ic} is a workable fracture criterion for higher strength, lower toughness materials, see chapter 5, section 5.2. The specimen sizes required for a valid K_{Ic} are convenient to handle for these materials. For lower strength, higher toughness materials K_{Ic} cannot be measured so conveniently because the specimen size required for a valid test may be prohibitively large. However, Landes (see reference 11) argued that the assumption that a K_{Ic} always can be measured for any material provided that a large enough specimen is used is not true. He showed that for some materials it is impossible to measure a valid K_{Ic}.

For ductile materials, *i.e.* materials that exhibit stable crack extension prior to failure, the K_{Ic} is defined at the point where the stable crack extension Δa is 2% of the original specimen crack size a. The specimen size requirement in terms of crack length is given by

$$a \geq 2.5 \left(\frac{K_{Ic}}{\sigma_{ys}}\right)^2 . \tag{7.32}$$

Combining this relation with $\Delta a = 0.02 \cdot a$ yields

$$\Delta a \geq 0.05 \left(\frac{K_{Ic}}{\sigma_{ys}}\right)^2 , \tag{7.33}$$

a relation which should be fulfilled to obtain a valid K_{Ic}.

To further examine the size requirement it is convenient to write K in terms of J, using equation (6.30). For arbitrary values of J equation (7.33) can be rewritten as

$$\Delta a \geq 0.05 \frac{E}{1-\nu^2} \frac{J}{\sigma_{ys}^2} \tag{7.34}$$

or

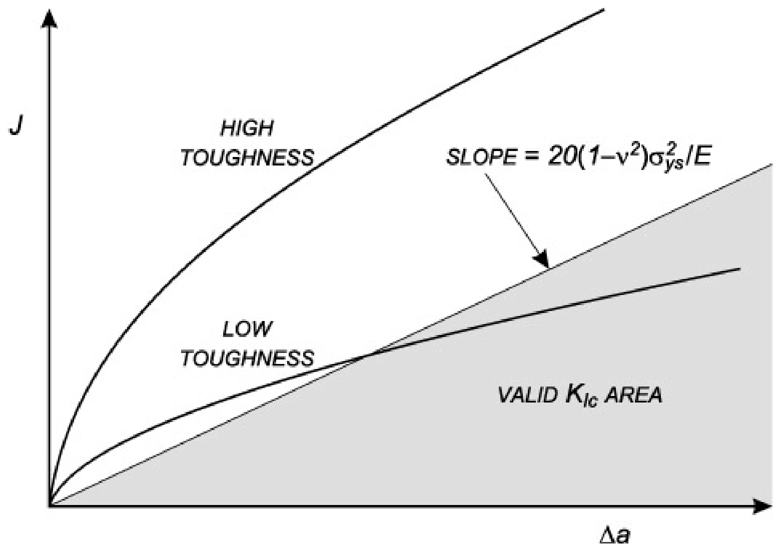

Figure 7.13. Schematic showing the K_{Ic} size requirement as an area in a J-Δa plot.

$$J \leq 20\,(1\text{-}\nu^2)\,\frac{\sigma_{ys}^2}{E}\,\Delta a\;. \qquad (7.35)$$

As a function of the absolute amount of crack extension Δa, this relation gives the maximum J value for which the K_{Ic} size requirement with respect to the crack length, equation (7.32), would be fulfilled. This condition is represented by the shaded area in the J-Δa plot of figure 7.13. Also, in this figure J resistance curves are schematically plotted for materials with a high and a low fracture toughness.

Irrespective of specimen size, a valid K_{Ic} for a certain material can only be obtained if for some crack extension the J resistance curve enters the shaded area. The required specimen size then follows from equating the crack extension at which this occurs to 2% of the initial crack length. Clearly, for the tougher material K_{Ic} cannot be determined no matter how large a specimen is used. For the material with the lower toughness, if all other requirements are fulfilled also (see section 5.2), a valid K_{Ic} value can be determined, albeit that sometimes unrealistic specimen sizes would be required.

For aluminium alloys and for high strength steels the K_{Ic} size requirement will be fulfilled. However for lower strength, higher toughness steels this certainly will not be the case: no valid K_{Ic} can be determined, regardless the specimen size.

7.6 The Standard $\delta_{t_{crit}}$ Test

At the beginning of this chapter it was remarked that the original $\delta_{t_{crit}}$ test was the subject of an official British Standard. At present the most recent version, designated BS 7448, dates from 1991, see reference 12 of the bibliography.

The Standard COD Specimens

The standard COD test specimens conform to the three-point notched bend (SENB) and the compact tension (CT) configurations already described in section 5.2. For CT specimens a J_{Ic} type starter notch is allowed also (see figure 7.7). The preferred W/B ratio is 2, but deviation is allowed within certain limits. In principle the thickness B must be equal to that of the material as used in service, and the specimens are *not* side grooved. Exceptions are allowed if it can be shown that a lesser thickness does not affect fracture toughness or if a relation between thickness and fracture toughness can be established.

It is important to note that the $\delta_{t_{crit}}$ values resulting from this test method may be affected by the specimen geometry and size. Therefore caution is required when comparing results from different sources.

Expressions for Calculating δ_t

Direct measurement of δ_t at the crack tip is impossible. Instead a clip gauge is used to measure the crack opening displacement, v_g, at or near the specimen surface. It is then assumed that the ligament b ($= W - a$) acts as a plastic hinge. This implies a rotation point within the ligament at some distance $r{\cdot}b$.

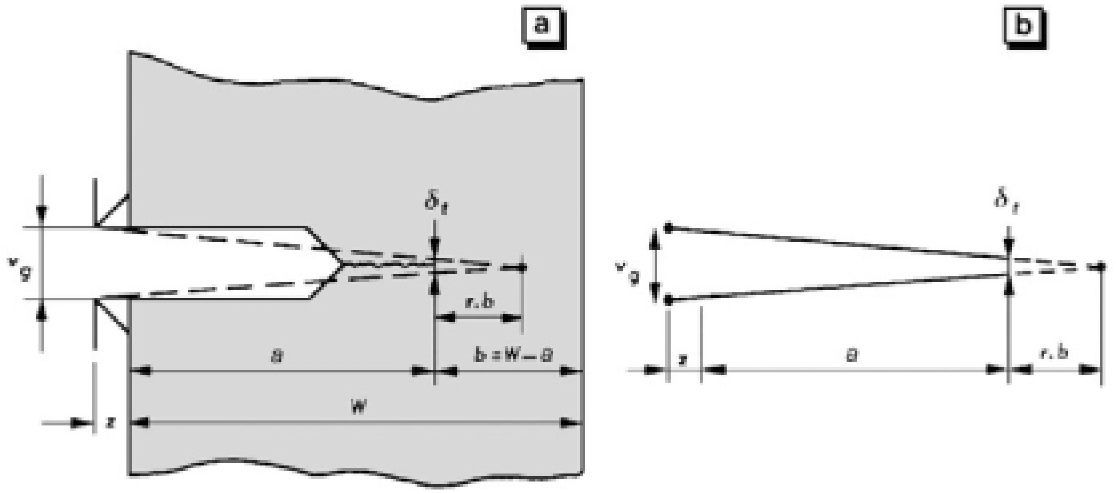

Figure 7.14. Relation between crack opening displacement v_g and crack tip opening displacement δ_t.

In figure 7.14.a an example is shown where the clip gauge is mounted on attachable knife edges on the specimen surface. Figure 7.14.b shows that δ_t can be expressed as

$$\delta_t = \frac{r \cdot b}{r \cdot b + a + z} v_g \,, \tag{7.36}$$

where the distance z corrects for the use of knife edges. In general $a + z$ should be interpreted as the distance between the position of the clip gauge and the crack tip. This possibly includes the size of attachable knife edges (see figure 5.5) and for CT specimens also depends on the type of notch used.

Although equation (7.36) is simple, there are two notable difficulties:

1) The value of the rotation factor r. Experiments show significant spread in the value to be used for r. This is because the determination requires complicated techniques, *e.g.* the double clip gauge method (reference 13) or infiltration of the crack with plastic or silicone rubber (reference 14). For the standard COD test the assigned r values are 0.4 for the SENB specimen and 0.46 for the CT specimen.
2) Interpretation of the clip gauge displacement v_g. The increase in v_g with loading from a null point setting is caused by two effects, namely elastic opening of the crack and rotation around $r \cdot b$. Thus to consider v_g as arising only from rotation, as in equation (7.36), would lead to erroneous results. Instead v_g must be separated into an elastic part v_{el} and a plastic part v_{pl} as shown schematically in figure 7.15.

Only the plastic part of the displacement is substituted into equation (7.36), *i.e.*

$$\delta_{pl} = \frac{v_{pl} \cdot r \cdot b}{r \cdot b + a + z} \,. \tag{7.37.a}$$

For reasons of accuracy the elastic part v_{el} is not used but the elastic contribution to δ_t is calculated according to the LEFM expression for CTOD, equation (3.20), modified for plane strain and a plastic constraint factor $C = 2$ (see also section 3.5), *i.e.*

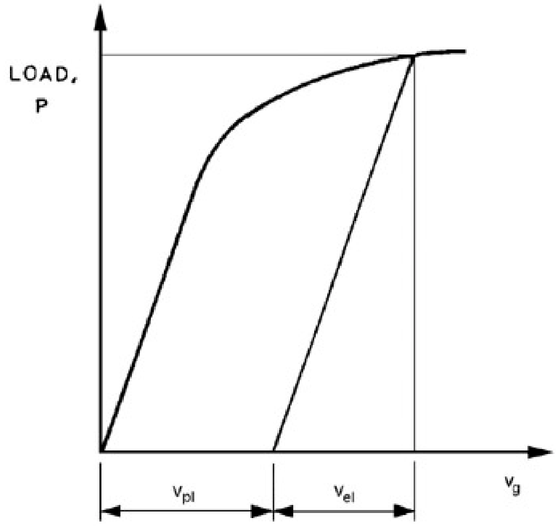

Figure 7.15. Separation of total crack opening displacement v_g into elastic (v_{el}) and plastic (v_{pl}) components.

$$\delta_{el} = \frac{K_I^2}{E\sigma_{ys}}\left(\frac{1-\nu^2}{2}\right). \tag{7.37.b}$$

and

$$\delta_t = \delta_{el} + \delta_{pl} = \frac{K_I^2(1-\nu^2)}{2E\sigma_{ys}} + \frac{r \cdot b}{r \cdot b + a + z} v_{pl} . \tag{7.38}$$

Note that the value of K_I in equation (7.38) is obtained from the standard formula for the SENB and CT specimens, equations (5.1) and (5.2), by substituting the initial crack length, *a*, and the load at which v_{pl} is measured.

As will be seen under the subheading "Analysis of Load-Displacement Records to Determine $\delta_{t_{crit}}$" several values of $\delta_{t_{crit}}$ can be defined. Note that the British Standard defines δ_t as the crack opening at the original crack tip, as shown in figure 6.7. This means that it is taken for granted that during loading the crack tip will displace and move forward owing to blunting, since at the very tip δ_t must always be zero.

COD Test Procedure

The steps involved in setting up and conducting a COD test are:

1) Prepare shop drawings of the specimen.
2) Specimen manufacture.
3) Fatigue precracking.
4) Obtain test fixtures and clip gauge for crack opening displacement measurement.
5) Testing.
6) Analysis of load-displacement records to determine $\delta_{t_{crit}}$.

Steps (1), (2) and (4) will not be considered further in view of previous discussions in section 5.2. Steps (3) and (5) will be reviewed here and step (6) will be dealt with under the next subheading.

The configuration of the starter notch for fatigue precracking is similar to that for the standard K_{Ic} specimens, see section 5.2, except that a straight notch is recommended rather than a chevron. Fatigue precracking has to be done with a stress ratio R (= $\sigma_{min}/\sigma_{max}$) between 0 and 0.1. As was the case for J_{Ic} testing, the maximum fatigue load should not exceed 40% of the plastic collapse load given in equations (7.27) and (7.28) for SENB and CT specimens respectively. These requirements are to ensure a sufficiently sharp precrack with limited residual plastic strain in the crack tip region.

During the actual COD test the specimen is loaded under displacement control while recording load and crack opening displacement. The test can be carried out with any testing machine incorporating a load cell to measure force electrically. The British Standard specifies that the loading rate should be such that the increase in stress intensity factor with time, dK_I/dt, is between 0.5 and 3.0 $MPa\sqrt{m}/s$. This is arbitrarily defined as 'static' loading, in the same way as for K_{Ic} testing. Again note that equations (5.1) or (5.2) may be used to calculate stress intensity factors.

Since the increase rate dK_I/dt is measured in the elastic region of the load-displacement curve this procedure can lead to large differences in loading rate for ductile specimens: if the loading rate of the testing machine is kept constant the rate of displacement will strongly increase in the plastic region of the load-displacement curve; if, on the other hand, the displacement rate of the testing machine is kept constant, the loading rate will decrease in the plastic region. It has been shown that low loading rates in the plastic region of the load-displacement diagram may lead to lower CTOD values, see reference 15 of the bibliography.

After the test the fracture surface must be examined. The procedure to determine the fatigue precrack length and the requirements that must be met to obtain a valid test result are the same as in J_{Ic} testing, see section 7.4. Furthermore, it is necessary to establish whether stable crack extension occurred during the test and to assess the amount of crack extension associated with possible pop-in behaviour, *i.e.* a small amount of unstable crack growth followed by crack arrest.

Analysis of Load-Displacement Records to Determine $\delta_{t_{crit}}$

The load-displacement records can assume six different forms. These are given schematically in figure 7.16. The assessment of $\delta_{t_{crit}}$ for each case will be briefly discussed.

Before classifying the measured load-displacement curve, it is necessary to decide whether possible pop-in behaviour must be considered significant. In all cases a pop-in is significant if post-test examination of the fracture surface reveals that the corresponding crack extension exceeded 4% of the uncracked ligament, b. Otherwise, a pop-in is only considered significant if at subsequent crack arrest the specimen compliance has dropped by more than 5%. A procedure for deciding this is suggested in the standard.

Cases 1, 2 and 3 are treated similarly. Cases 1 and 2 are monotonically rising load-displacement curves showing no or limited plasticity and no stable crack extension before fracture. Case 3 shows a (significant) pop-in owing to sudden crack extension and arrest. In all these cases $\delta_{t_{crit}}$ is taken to be δ_c, which is calculated according to equation (7.38) using P_c and v_c.

Cases 4 and 5 may also be treated similarly. Prior to instability, which again is either fracture or a (significant) pop-in, stable crack extension occurs. This should be revealed after the test by examination of the fracture surface. In these cases $\delta_{t_{crit}}$ is calculated as δ_u at (P_u,v_u).

Case 6 is relevant to extremely ductile materials for which stable crack extension proceeds beyond maximum load P_m: $\delta_{t_{crit}}$ is calculated as δ_m corresponding to (P_m,v_m).

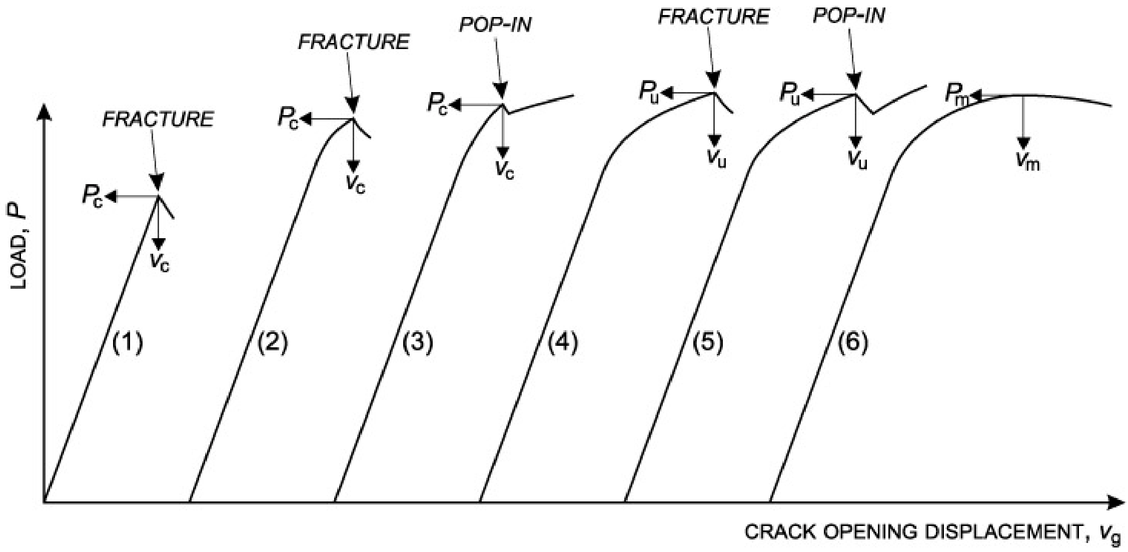

Figure 7.16. Types of load - crack opening displacement plots obtained during COD testing.

Concluding Remarks

The significance of $\delta_{t_{crit}}$ is somewhat limited in practice. Materials can be classified with it and to a certain extent $\delta_{t_{crit}}$ can be used in failure assessment procedures (see section 8.2). However, test results cannot be used to assess the effect of stable crack growth on crack resistance. For this purpose the British Standard Institution has published additional standards. These bear more resemblance to the J_{Ic} test procedure described in section 7.4.

7.7 Bibliography

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